01-24-2019

Integrals with trig. functions (cont'd)

Ex:

- $\int \sin^6(x)\cos^3(x)\,dx$ $\qquad\qquad \sin^2x + \cos^2x = 1$

Rule of thumb: look at the odd coefficient/exponent. One will go away with "du" and the remaining even can be converted.

$u = \sin(x) \qquad du = \cos(x)\,dx \qquad dx = \frac{du}{\cos(x)}$

$= \int \sin^6(x)\,(1-\sin^2(x))\cos(x)\,dx$

$= \int u^6(1-u^2)\cancel{\cos(x)}\frac{du}{\cancel{\cos(x)}}$

$= \int u^6 - u^8\,du = \frac{1}{7}u^7 - \frac{1}{9}u^9 + c$

$= \frac{1}{7}\sin^7(x) - \frac{1}{9}\sin^9(x) + C$

If both of your exponents are odd, you can pick either one, but one will be easier.
If both are even, then this technique doesn't work.

Ex:

- $\int \sin^2(x)\cos^2(x)\,dx$ There are 3 possible ways to work this out.

For this, we'll need to review some trig. formulas:

(1) $\cos^2(x) = \frac{1+\cos(2x)}{2}$ $\qquad \sin^2(x) = \frac{1-\cos(2x)}{2}$ (2)

(3) $\sin(2x) = 2\sin(x)\cos(x)$ → we'll use this one

The following trick applies only when the power/expon. is the same for both

$= \int (\sin(x)\cos(x))^2\,dx = \int \left(\frac{1}{2}\sin(2x)\right)^2 dx$

$= \frac{1}{4}\int \sin^2(2x)\,dx$ We'll now use (2)

$= \frac{1}{4}\int \left(\frac{1-\cos(4x)}{2}\right)dx = \frac{1}{8}\int 1 - \cos(4x)\,dx = \frac{1}{8}\left(x - \frac{1}{4}\sin(4x)\right) + C$

- $\int \sin^2(x)\cos^2(x)\,dx$ Let's now pick a different path.

$= \int \frac{1}{2}(1-\cos(2x))\frac{1}{2}(1+\cos(2x))\,dx = \frac{1}{4}\int 1^2 - \cos^2(2x)\,dx$ Now use (1) Or replace it with $\sin^2(x)$

$\frac{1}{4}\int 1 - \frac{1}{2}(1+\cos(2x))\,dx$ → and you solve it

$\frac{1}{4}\int \sin^2(2x)\,dx$ use (2) → and you solve it

Beware: Sometimes picking different paths will take you to different results.

Ex:

Remember: $\boxed{\tan^2(x) + 1 = \sec^2(x)}$

- $\int \sec^9(x)\tan^5(x)\,dx$

$u = \sec(x) \qquad du = \sec(x)\tan(x)\,dx \qquad dx = \frac{du}{\sec x \cdot \tan x}$

$= \int \sec^8(x)\tan^4(x) \cdot \tan(x)\sec(x)\,dx$

$= \int \sec^8(x)\left(\tan^2(x)\right)^2 \tan(x)\sec(x)\,dx$

$= \int \sec^8(x)\left(\sec^2(x) - 1\right)^2 \tan(x)\sec(x)\,dx$

$= \int u^8(u^2-1)^2\,du = \int u^8(u^4+1-2u^2)\,du = \int u^{12} + u^8 - 2u^{10}\,du$

$= \frac{1}{13}u^{13} + \frac{1}{9}u^9 - \frac{2}{11}u^{11} + C$

$= \frac{1}{13}\sec^{13}(x) + \frac{1}{9}\sec^9(x) - \frac{2}{11}\sec^{11}(x) + C$