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Integration by Parts

Example:

- $$\int_{-1}^{2} xe^{6x}\,dx = \frac{1}{6}xe^{6x} - \frac{1}{36}e^{6x}\Big|_{-1}^{2}$$ Definite integrals won't show up often in class.
$$= \frac{11}{36}e^{12} + \frac{7}{36}e^{-6}$$

Remember:
$$\int u\,dv = uv - \int v\,du$$

Ex:

$u = 3t+5$ $\quad dv = \cos(\frac{t}{4})$
$du = 3$ $\quad v = 4\sin(t/4)$

- $$\int (3t+5)\cos(\tfrac{t}{4})\,dt$$
$$= (3t+5)4\sin(t/4) - 12\int \sin(t/4)\,dt$$
$$= (3t+5)4\sin(t/4) + 48\cos(t/4) + C$$

Ex:

$u = w^2$ $\quad dv = \sin(10w)$
$du = 2w$ $\quad v = \frac{-1}{10}\cos(10w)$

- $$\int w^2\sin(10w)\,dw$$
$$= \frac{-1}{10}w^2\cos(10w) + \frac{1}{5}\int w\cos(10w)\,dw$$
integ. by parts again!

$u = w$ $\quad dv = \cos(10w)$
$du = 1$ $\quad v = \frac{1}{10}\sin(10w)$

$$= \frac{-1}{10}w^2\cos(10w) + \frac{1}{5}\left[\frac{1}{10}w\sin(10w) - \frac{1}{10}\int\sin(10w)\,dw\right]$$
$$= \frac{-1}{10}w^2\cos(10w) + \frac{1}{5}\left[\frac{1}{10}w\sin(10w) + \frac{1}{100}\cos(10w)\right]$$
$$= \frac{-1}{10}w\cos(10w) + \frac{1}{50}w\sin(10w) + \frac{1}{500}\cos(10w) + C$$

Ex:

$u = \ln(x)$ $\quad dv = 1$
$du = 1/x$ $\quad v = x$

- $$\int \ln(x)\,dx$$
$$= x\ln(x) - \int \frac{1}{x}\cdot x\,dx = x\ln(x) - \int 1\,dx = x\ln(x) - x + C$$

Ex:

$u = \sin\theta$ $\quad dv = e^\theta$
$du = \cos\theta$ $\quad v = e^\theta$

- $$\int e^\theta \sin\theta\,d\theta$$
$$= e^\theta\sin\theta - \int e^\theta\cos\theta\,d\theta$$
integ. by parts again!

$u = \cos\theta$ $\quad dv = e^\theta$
$du = -\sin\theta$ $\quad v = e^\theta$

$$= e^\theta\sin\theta - \left[e^\theta\cos\theta + \int e^\theta\sin\theta\,d\theta\right]$$
$$\int e^\theta\sin\theta\,d\theta = e^\theta\sin\theta - e^\theta\cos\theta - \int e^\theta\sin\theta\,d\theta$$
$$\int e^\theta\sin\theta\,d\theta + \int e^\theta\sin\theta\,d\theta = e^\theta\sin\theta - e^\theta\cos\theta$$
$$2\int e^\theta\sin\theta\,d\theta = e^\theta(\sin\theta - \cos\theta)$$
$$\int e^\theta\sin\theta\,d\theta = \frac{1}{2}e^\theta(\sin\theta - \cos\theta) + C$$

Integrals w/trig functions:

Let's start with a Calc. 1 problem to review our skills.

- $\int \cos(x)\sin^5(x)\,dx$, which we do by u-substitution.

  $u = \sin(x) \quad du = \cos(x)\,dx \quad dx = \frac{du}{\cos(x)}$

  $= \int \cancel{\cos(x)}\, u^5 \frac{du}{\cancel{\cos(x)}}$

  ... and we can continue it easily.

But what if the problem is like this:

$\int \sin^5(x)\,dx = \int \sin(x)\sin^4(x)\,dx$ has to be even

has to be odd

Now, remember: $\boxed{\sin^2(x) = 1 - \cos^2(x)}$

$= \int \sin(x)(\sin^2(x))^2\,dx$

$= \int \sin(x)(1-\cos^2(x))^2\,dx$

Now, u-substitution $u = \cos(x) \to du = -\sin(x)dx \to dx = \frac{du}{-\sin(x)}$

$= \int \cancel{\sin(x)}(1-u^2)^2 \frac{du}{-\cancel{\sin(x)}} = -\int (1-u^2)^2\,du$

$= -\int (1+u^4-2u^2)\,du \quad [\ldots]$

$= -\cos(x) + \frac{2}{3}\cos^3(x) - \frac{1}{5}\cos^5(x) + C$ //