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# Calculus 2: Introduction

## Integration by parts:

Review:

- $\int e^x dx = e^x + C \longrightarrow$ always remember it!

What do indefinite integ. ask us for? What fxn did we differentiate to get "$e^x$".

- $\int e^{6x} dx = \frac{1}{6} e^{6x} + C$

  or by substitution

  $\int e^{6x} dx$, let $u = 6x \rightarrow du = 6dx \longrightarrow \frac{du}{6} = dx$

  then $\int e^u \frac{du}{6} = \frac{1}{6}\int e^u du = \frac{1}{6} e^u + C \underset{u=6x}{=} \frac{1}{6} e^{6x} + C$

- $\int x e^{x^2} dx = \frac{1}{2} e^{x^2} + C$

  let $u = x^2$, $du = 2x\,dx$, $dx = \frac{du}{2x}$, then $\int x e^u \frac{du}{2x} = \frac{1}{2}\int e^u du \underset{u=x^2}{=} \frac{1}{2} e^{x^2} + C$

- $\int x e^{6x} dx$. Here's where we do integ. by parts!
  U-sub is useless here.

How and why?

$[f(x)g(x)]' = [fg]' = f'g + fg'$ Product rule

integrate $\int [fg]' dx = \int f'g + fg' dx$

$f \cdot g = \int f'g\,dx + \int fg' dx$

$\int f(x) g'(x) dx = f(x)g(x) - \int f'(x) g(x) dx$

Integration by parts

But this other formula is more popular:

let $u = f(x)$, $v = g(x)$, $du = f'(x)dx$, $dv = g'(x)dx$

$$\boxed{\int u\,dv = uv - \int v\,du}$$

How to use it though?

*example:* $\int x e^{6x}\, dx$ We'll learn to figure out what's "u" and what's "v".

$u = x \qquad dv = e^{6x}\, dx$

$du = dx \qquad v = \int dv = \int e^{6x}\, dx = \frac{1}{6} e^{6x}$

Now use integ. by parts formula!

$$\int x e^{6x}\, dx = \frac{1}{6} x e^{6x} - \frac{1}{6} \int e^{6x}\, dx$$

Now this is easy to do!

$$= \frac{1}{6} x e^{6x} - \frac{1}{36} e^{6x} + C$$ → at the end always!