Logarithms

Recall: $\ln x = \int_1^x \frac{1}{t}\,dt \quad , \quad x>0$

1. Domain : $(0,\infty)$ vert asymptote : $x=0$

   Range : $(-\infty,\infty)$ no horiz asymptote

2. $D[\ln x] = \frac{1}{x}$

   $D^2[\ln x] = \frac{-1}{x^2}$

   $\ln x$ is increasing and concave down

3. $\ln 1 = 0$

   e is the number such that $\ln e = 1$

   $\ln x > 0$ for $x > 1$

   $\ln x < 0$ for $0 < x < 1$

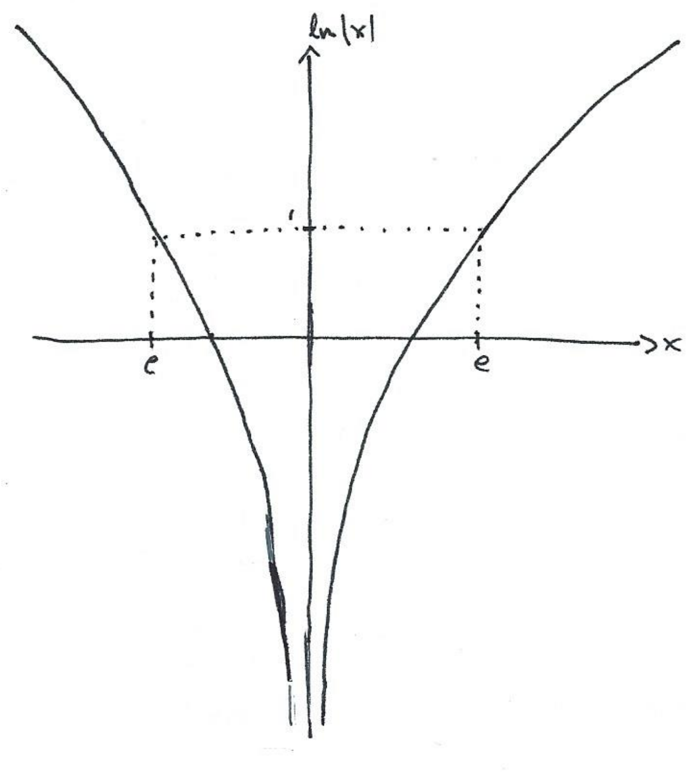
Consider $\ln|x|$ Domain : all reals except 0

$$\ln|x| = \begin{cases} \ln(x), & x>0 \\ \ln(-x), & x<0 \end{cases}$$

$$D[\ln|x|] = \begin{cases} D[\ln(x)], & x>0 \\ D[\ln(-x)], & x<0 \end{cases} = \begin{cases} \frac{1}{x}, & x>0 \\ \frac{1}{x}, & x<0 \end{cases}$$

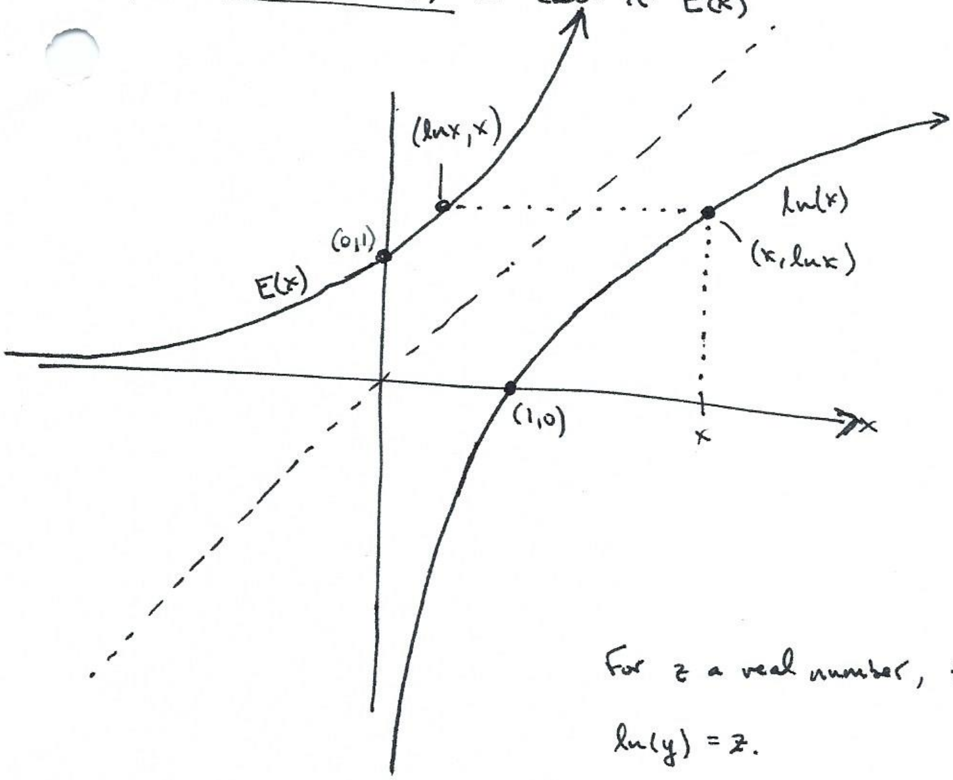
$$D[\ln(-x)] = \frac{1}{-x} D[-x] = \frac{1}{x}$$

for $x \neq 0$, $\int \frac{1}{x}\,dx = \ln|x| + C$



EXPONETIAL FUNCTION

Inverse of $\ln(x)$ – call it $E(x)$



$E(x)$:
- Domain : $(-\infty,\infty)$
- Range : $(0,\infty)$
- $E(0)=1$
- horiz asym : $y=0$
- no vert asymp.
- $E(x)$ increasing, concave up, continuous

For z a real number, there is exactly one number y s.t. $\ln(y)=z$.

$\ln(e^z) = z\ln(e) = z\cdot(1) = z \mapsto \ln(e^z)=z$.

Then $y=e^z$.

☆ $E(x) = e^x = \exp(x)$

$y=e^x$ and $y=\ln x$ are inverses of each other

$\rightarrow$ $x>0$, $e^{\ln x} = x$

$x\in\mathbb{R}$, $\ln(e^x)=x$

What is $D(e^x)$?

for $x\in\mathbb{R}$,

$\ln(e^x) = x$

take derivative $\frac{1}{e^x}D(e^x) = 1$

* $D(e^x) = e^x$ *

*

Show that $e^{x+y} = e^x e^y$

$\ln(e^{x+y}) = (x+y)\ln(e) = x+y$

$\ln(e^x e^y) = \ln(e^x) + \ln(e^y) = x\ln(e) + y\ln(e)$
$= x+y$

$\ln(e^{x+y}) = \ln(e^x e^y) \Rightarrow e^{x+y} = e^x e^y$.

Exponential Function

Ex. $\int 6e^{3x}\,dx = 6\int e^{3x}\,dx = \frac{6}{3}\int e^{u}\,du = 2e^{u}+C = 2e^{3x}+C$

$u = 3x$

$du = 3dx$

Ex. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = 2\int e^{u}\,du = 2e^{u}+c = 2e^{\sqrt{x}}+c$

$u = \sqrt{x}$

$du = \frac{dx}{2\sqrt{x}}$

Ex. $\int \frac{e^{3x}}{e^{3x}+1}\,dx = \frac{1}{3}\int \frac{du}{u+1} = \frac{1}{3}\int \frac{dw}{w} = \frac{1}{3}\ln|w|+c = \frac{1}{3}\ln|u+1|+C = \frac{1}{3}\ln|e^{x}+1|+C$

$u = e^{3x}$ $\quad$ $w = u+1$

$du = 3e^{3x}dx$ $\quad$ $dw = du$

$\frac{1}{3}du = e^{3x}dx$

Ex. $D[xe^{x}] = D[x]e^{x} + x\,D[e^{x}] = e^{x} + xe^{x}$

Ex. $D[e^{x^2}] = e^{x^2}D[x^2] = 2xe^{x^2}$

Ex. $D[\ln(x^2)] = D[2\ln x] = 2\,D[\ln x] = \frac{2}{x}$

$= \frac{1}{x^2}D[x^2] = \frac{2x}{x^2} = \frac{2}{x}$

Ex. $D[(\ln x)^2] = 2\ln(x)\,D[\ln x] = \frac{2\ln(x)}{x}$

Ex. $D[\ln(e^{x}+x^2)] = \frac{D[e^{x}+x^2]}{e^{x}+x^2} = \frac{e^{x}+2x}{e^{x}+x^2}$