LOGARITHMS

Proof (cont). (show true for neg integers)

(-1) we already know $\ln\left(\frac{1}{x}\right) = -\ln(x) = \ln(x^{-1})$ ✓

Then if $(-n)$ is a negative integer, then

$$\ln(x^{-n}) = \ln\left((x^{-1})^n\right) = n\ln(x^{-1}) = -n\ln(x)$$

(show true for 0)

$$\ln(x^0) = \ln(1) = 0 = 0 \cdot \ln(x)$$

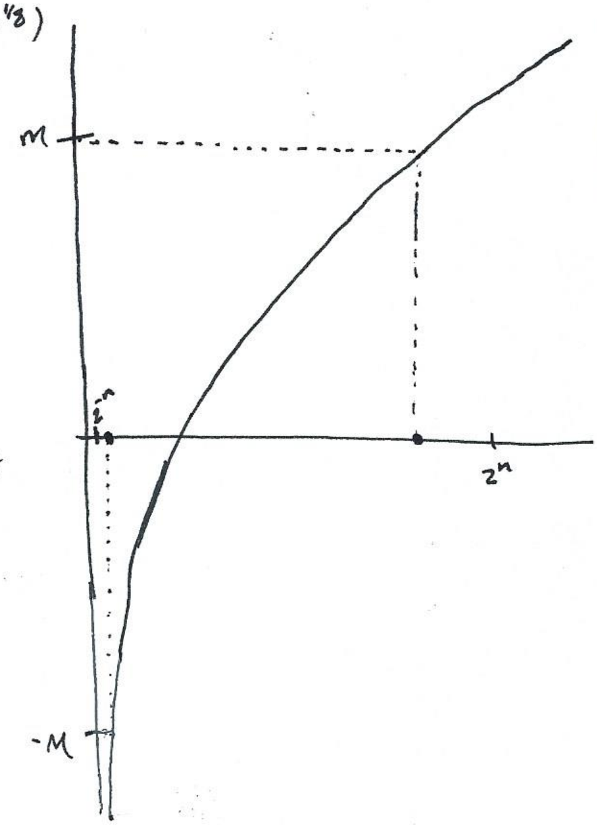
(show true for rational numbers $r = \frac{p}{q}$, where p, q integers and $q \neq 0$)

i) Show $\ln(x^{p/q}) = \frac{p}{q}\ln(x)$

$x = (x^{1/q})^q$

$\ln(x) = \ln\left[(x^{1/q})^q\right] = q\ln(x^{1/q}) \longrightarrow \frac{1}{q}\ln(x) = \ln(x^{1/q})$

Then $\ln\left[(x^p)^{1/q}\right] = \frac{1}{q}\ln(x^p) = \frac{p}{q}\ln(x)$



What is the range of $\ln x$?

Suppose $M > 0$.

$\ln 2 = \int_1^2 \ln x \, dx > 0$. Then there is some pos. int. n such that

$n\ln 2 > M$. $\cdots\cdots\Rightarrow$ $-n\ln 2 < M$

$n\ln 2 = \ln(2^n) > M$ $\qquad \ln(2^{-n}) < M$

LOGARITHMS

Math 2413
Dr. Kennedy
8 Apr 2019
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Examples

$$\int_1^3 \frac{1}{x}\,dx = \ln x \Big|_1^3 = \ln 3 - \ln 1 = \ln 3$$

$$\int \frac{1}{x^2}\,dx = \frac{x^{-2+1}}{-2+1} + C = -x^{-1} + C$$

$$\int \frac{x}{x^2+1}\,dx = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| + c = \frac{1}{2}\ln(x^2+1) + C$$

$u = x^2+1$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$$D\left[\ln(x\sqrt{x+1})\right] \xrightarrow[\text{inside: } x\sqrt{x+1}]{\text{outside: } \ln u} = \frac{1}{u}D\left[x\sqrt{x+1}\right] = \frac{1}{x\sqrt{x+1}}\left[\sqrt{x+1} + \frac{x}{2\sqrt{x+1}}\right]$$