Math 2413
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# Indefinite Integrals — Substitution

**Ex.** $\int (2x+1)^{1/2}\,dx = \frac{1}{2}\int u^{1/2}\,du = \frac{1}{2}\frac{u^{3/2}}{3/2} + C = \frac{1}{3}u^{3/2} + C = \frac{1}{3}(2x+1)^{3/2} + C$

$u = 2x+1$

$du = 2\,dx$

$\frac{1}{2}du = dx$

For definite integrals, we have to change our endpoints, too. If we do this, then

$\int_0^1 (2x+1)^{1/2}\,dx = \frac{-1}{8}\int_1^3 u^{1/2}\,du = \ldots = \frac{1}{3}u^{3/2}\Big|_1^3 = \frac{1}{3}\left(3^{3/2} - 1\right)$

$u(0) = 1$

$u(1) = 3$

$= \frac{1}{3}(2x+1)^{3/2}\Big|_0^1 = \frac{1}{3}\left[(2+1)^{3/2} - (1)^{3/2}\right] = \frac{1}{3}\left(3^{3/2} - 1\right)$

**Ex.** $\int \frac{x}{(1-4x^2)^{1/2}}\,dx = \frac{-1}{8}\int \frac{du}{u^{1/2}} = -\frac{1}{8}\int u^{-1/2}\,du = -\frac{1}{8}\frac{u^{1/2}}{1/2} + C = -\frac{1}{4}u^{1/2} + C = -\frac{1}{4}(1-4x^2)^{1/2} + C$

$u = 1-4x^2$

$du = -8x\,dx$

$-\frac{1}{8}du = x\,dx$

**Ex.** $\int \sin^2(x)\cos(x)\,dx = \int u^2\,du = \frac{1}{3}u^3 + C = \frac{1}{3}\sin^3(x) + C$

$u = \sin x$

$du = \cos x\,dx$

**Ex.**