

# Integrals

Math 2413  
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$\int_a^b f(x) dx$  ← definite integral, is a number

$\int f(x) dx$  ← indefinite integral (antiderivative), is a family of functions

One technique used to make complicated integrals simpler is u-substitution (or just substitution).

Ex.  $\int 2x(1+x^2)^{1/2} dx = \int \underbrace{(1+x^2)^{1/2}}_{u^{1/2}} \underbrace{2x dx}_{du} = \int u^{1/2} du = \frac{2}{3} u^{3/2} + C = \frac{2}{3} (1+x^2)^{3/2} + C$

let  $u = 1+x^2$   
 $\frac{du}{dx} = 2x$   
 $du = 2x dx$

Check:  $D\left[\frac{2}{3}(1+x^2)^{3/2} + C\right]$

$= \frac{2}{3} \cdot \frac{3}{2} (1+x^2)^{1/2} (2x) + 0$

$= 2x(1+x^2)^{1/2} \checkmark$

\* we are doing  
chain rule backwards

Ex.  $\int x^3 \cos(x^4+2) dx = \int \underbrace{\cos(x^4+2)}_{\cos u} \underbrace{x^3 dx}_{\frac{1}{4} du} = \frac{1}{4} \int \cos(u) du = \frac{1}{4} \sin(u) + C = \frac{1}{4} \sin(x^4+2) + C$

$u = x^4 + 2$

$\frac{du}{dx} = 4x^3$

$du = 4x^3 dx$

$\frac{1}{4} du = x^3 dx$

Ex.  $\int \cos(x) [\sin(x)+10]^{1/2} dx = \int u^{1/2} du =$

$u = \sin x + 10$

$du = \cos x dx$