Math 2413
Dr. Kennedy
15 Apr 2019
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Find f'

Ex

$f(x) = x^x = e^{x\ln x} \mapsto D(x^x) = D(e^{x\ln x}) = e^{x\ln x} D(x\ln x)$

$x > 0$

$= x^x D(x\ln x) = x^x\left[\ln x + x\left(\frac{1}{x}\right)\right] = x^x(\ln(x) + 1)$

logarithmic differentiation:

Let $y = x^x$. Then $\ln y = \ln(x^x) = x\ln x$

So $\ln y = x\ln x$. Now use implicit differentiation

$\frac{1}{y}y' = \ln x + x\left(\frac{1}{x}\right) = \ln x + 1$

$\frac{1}{y}y' = \ln x + 1 \mapsto y' = y[\ln x + 1] = x^x[\ln x + 1]$

Ex $f(x) = x^{\sin x} = e^{\sin(x)\ln(x)} \mapsto f'(x) = e^{\sin(x)\ln(x)}\left(\sin(x)\ln(x)\right)'$

$= e^{\sin(x)\ln(x)}\left(\cos(x)\ln(x) + \sin(x)\left(\frac{1}{x}\right)\right)$

$= x^{\sin(x)}\left(\cos(x)\ln(x) + \frac{1}{x}\sin(x)\right)$

logarithmic differentiation:

Let $y = x^{\sin x}$

$\ln y = \sin(x)\ln(x) \mapsto \frac{y'}{y} = \cos(x)\ln(x) + \sin(x)\left(\frac{1}{x}\right)$

$y' = y\left[\cos(x)\ln(x) + \frac{1}{x}\sin(x)\right]$

$= x^{\sin(x)}\left[\cos(x)\ln(x) + \frac{1}{x}\sin(x)\right]$

Graphing

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Ex $f(x) = xe^x$

Domain: $\mathbb{R}$.

x int : 0

y int : 0

no vert asymp (f is cont on $\mathbb{R}$)

horiz. asymp:

$\lim_{x \to \infty} xe^x = \infty$

$\lim_{x \to -\infty} xe^x = 0 \leftarrow$ HA @ $y=0$

$f'(x) = (1)e^x + x(e^x) = e^x(x+1)$

f' always defined

$0 = f'(x) = e^x(x+1) \Longrightarrow \underline{x=-1}$

only crit point of f

f': $-$ on the left of -1, $+$ on the right of -1

local min : $f(-1) = -\frac{1}{e}$

f dec on $(-\infty, -1)$

f inc on $(-1, \infty)$

$f''(x) = e^x(x+1) + e^x = e^x(x+2)$

f'' always defined

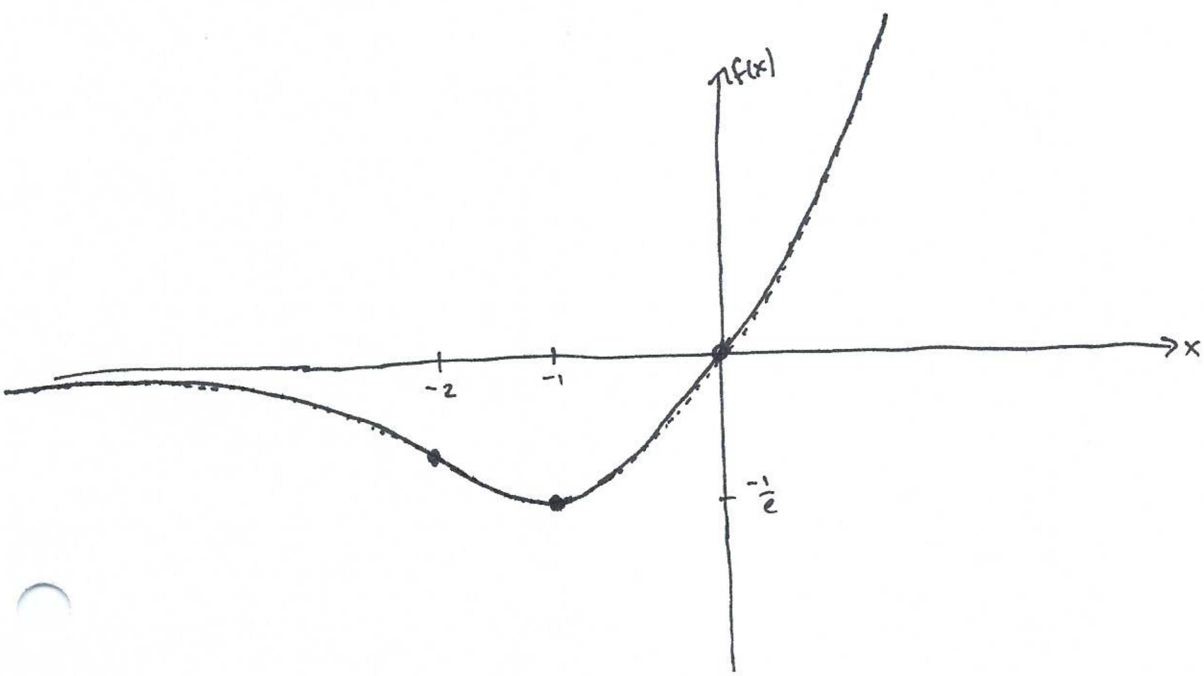
$0 = f''(x) = e^x(x+2) \Longrightarrow x = -2$

f'': $-$ on the left of -2, $+$ on the right of -2

infl. point : $(-2, f(-2)) = (-2, -2e^{-2})$

f conc down on $(-\infty, -2)$

f conc up on $(-2, \infty)$



Graphing

Ex. $f(x) = x^2 \ln x$

domain: $x > 0$

y int: none

x int: $0 = x^2 \ln x$

$\Rightarrow \ln x = 0$

$\Rightarrow x = 1$

hor. asy.: $\lim_{x \to \infty} x^2 \ln x = \infty \Rightarrow$ no HA

Ver. asy: only possibility: $x = 0$

$\lim_{x \to 0^+} x^2 \ln x = 0$

$f'(x) = 2x \ln x + x^2\left(\frac{1}{x}\right)$

$= 2x \ln x + x$

$= x(2\ln x + 1)$

f' cont on dom of f

$f' = 0 \Rightarrow x = 0$ or $2\ln x + 1 = 0$

$x = 0$ not in domain

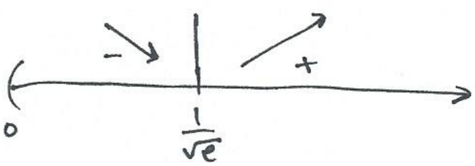
$\longrightarrow 2\ln x + 1 = 0$

$\ln x = -\frac{1}{2}$

$e^{\ln x} = e^{-\frac{1}{2}}$

$x = \frac{1}{\sqrt{e}}$

f'



local min: $f\left(\frac{1}{\sqrt{e}}\right) = \frac{1}{e}\ln\left(\frac{1}{\sqrt{e}}\right)$

$= \frac{1}{e}\left(-\frac{1}{2}\ln e\right)$

$= \frac{-1}{2e}$

f dec on $\left(0, \frac{1}{\sqrt{e}}\right)$

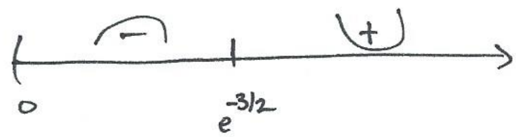
f inc on $\left(\frac{1}{\sqrt{e}}, \infty\right)$

$f''(x) = 2\ln x + 2x\left(\frac{1}{x}\right) + 1 = 2\ln x + 3$

$2\ln x + 3 = 0 \Rightarrow \ln x = -\frac{3}{2}$

$x = e^{-3/2}$

f''



infl. pt: $\left(e^{-3/2}, f(e^{-3/2})\right) = \left(e^{-3/2}, e^{-3}\ln(e^{-3/2})\right)$

$= \left(e^{-3/2}, -\frac{3}{2}e^{-3/2}\right)$

f conc down on $(0, e^{-3/2})$

f conc up on $(e^{-3/2}, \infty)$