EXP AND LOG

Evaluate

Ex. $\log_2(64) = \log_2(2^6) = 6\log_2(2) = 6(1) = 6$ $\quad$ * $\log_a(a) = 1 \iff a^1 = a$

Ex. $\ln(e^2) - \ln(e^4) = 2\ln(e) - 4\ln(e) = 2 - 4 = -2$

Ex. $\log_{32}(8) = \dfrac{\ln(8)}{\ln(32)} = \dfrac{\ln(2^3)}{\ln(2^5)} = \dfrac{3\ln(2)}{5\ln(2)} = \dfrac{3}{5}$

Solve

Ex. $\ln x = 3$

$e^{\ln x} = e^3$

$x = e^3$

Ex. $2\ln(x+2) - \frac{1}{2}\ln(x^4) = 1$

$\ln\left[(x+2)^2\right] - \ln(x^2) = 1$

$\ln\left[\dfrac{(x+2)^2}{x^2}\right] = 1$

$\ln\left[\left(\dfrac{x+2}{x}\right)^2\right] = 1$

$\left(\dfrac{x+2}{x}\right)^2 = e$

$\left(1 + \dfrac{2}{x}\right)^2 = e$

$1 + \dfrac{2}{x} = \pm\sqrt{e}$

$\dfrac{2}{x} = -1 \pm \sqrt{e}$

$\dfrac{x}{2} = \dfrac{1}{-1 \pm \sqrt{e}}$

$x = \dfrac{2}{-1 \pm \sqrt{e}}$

Exp and Log

Find domain and derivative:

Ex

$f(x) = \ln(x^3+1)$

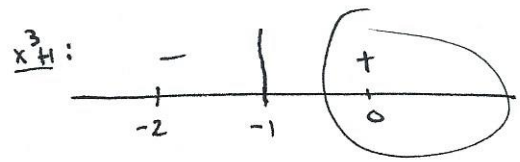
domain of $\ln x$ is positive real numbers

domain $f(x)$: $x^3+1>0 \rightarrow$ Find where $x^3+1=0$

$x^3=-1$

$x=-1$

x^3+1:



$\Longrightarrow$ domain of $f(x)$ is $(-1,\infty)$

$$Df(x) = D\ln(x^3+1) = \frac{1}{x^3+1}D(x^3+1) = \frac{3x^2}{x^3+1}$$

Ex.

$g(x) = \ln(\ln x) \longrightarrow x>0$ and $\ln x>0$, $\ln x>0 \Rightarrow x>1$

$\Longrightarrow$ domain of $g(x) = (1,\infty)$

$$Dg(x) = D\ln(\ln x) = \frac{1}{\ln x}D\ln x = \frac{1}{x\ln x}$$

Ex

$f(x) = \dfrac{e^{2x}-1}{e^{2x}+1}$

e^x is always positive, so $e^{2x}+1$ is never 0 $(e^{2x}+1>1)$

domain of $f(x)$ is $(-\infty,\infty)$

$$f'(x) = \left[\frac{e^{2x}-1}{e^{2x}+1}\right]' = \frac{2e^{2x}(e^{2x}+1)-(e^{2x}-1)(2e^{2x})}{(e^{2x}+1)^2} = \frac{2e^{2x}(e^{2x}-e^{2x}+1+1)}{(e^{2x}+1)^2} = \frac{4e^{2x}}{(e^{2x}+1)^2}$$

$$= \left(\frac{2e^x}{e^{2x}+1}\right)^2 = \left(\frac{2}{e^x+e^{-x}}\right)^2$$

Ex. $$\int \frac{x+1}{x^2}\,dx = \int\left[\frac{x}{x^2}+\frac{1}{x^2}\right]dx = \int\left(\frac{1}{x}+\frac{1}{x^2}\right)dx = \ln|x| - \frac{1}{x} + C$$

Exp and Log

Ex. $\int \frac{\log_2 x}{x}\,dx = \int \frac{1}{x}\frac{\ln x}{\ln 2}\,dx = \frac{1}{\ln 2}\int \frac{\ln x}{x}\,dx = \frac{1}{\ln 2}\int u\,du = \frac{u^2}{2\ln 2} + C$

$u = \ln x$

$du = \frac{1}{x}dx$

$= \frac{(\ln x)^2}{2\ln 2} + C$

Ex. $\int x\,2^{x^2}\,dx = \frac{1}{2}\int 2^u\,du = \frac{1}{2}\int e^{u\ln 2}\,du = \frac{1}{2\ln 2}e^{u\ln 2} + C = \frac{1}{2\ln 2}e^{x^2\ln 2} + C$

$u = x^2$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$= \frac{1}{2\ln 2}2^{x^2} + C = \frac{1}{\ln 2}2^{x^2-1} + C$