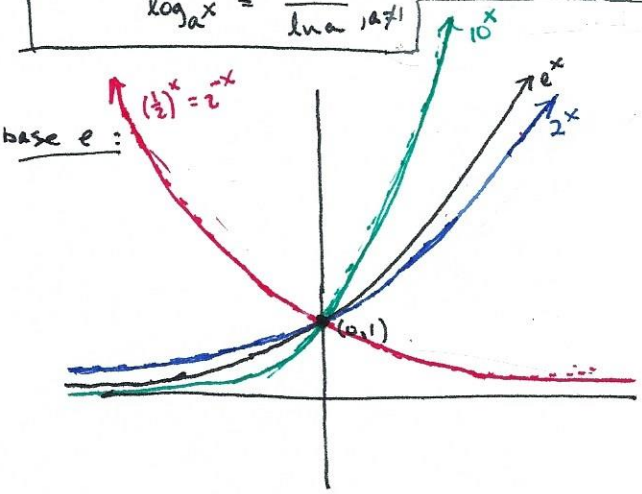
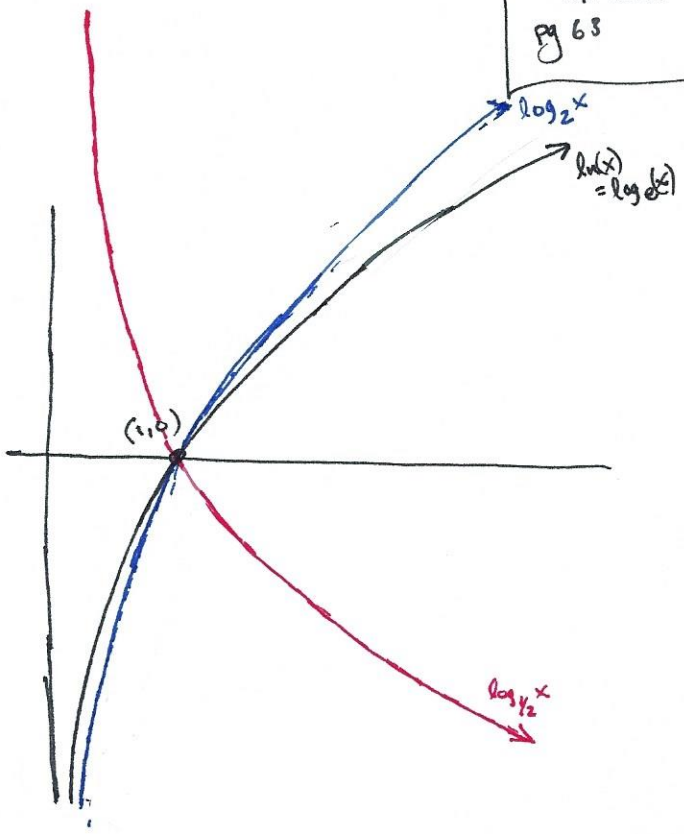
Exponentials and Logs with other bases

Define $a^x = e^{x \ln a}$ (a positive constant, x real variable)

$\log_a x = \frac{\ln x}{\ln a}$, $a \neq 1$





$2^x = e^{x \ln 2}$, $2 < e \approx 2.718$

x	2^x	2^{-x}
-2	$1/4$	4
-1	$1/2$	2
0	1	1
1	2	$1/2$
2	4	$1/4$

$D[a^x] = D[e^{x \ln a}] = e^{x \ln a} D[x \ln a] = e^{x \ln a} \ln a = a^x \ln a$

If $a = e$, then $D[a^x] = a^x \ln a = e^x \ln e = e^x$

This is what makes e the "natural" base.

Ex. $\int 2^x \, dx = \frac{2^x}{\ln 2} + C$ * since $D\left[\frac{2^x}{\ln 2}\right] = \frac{1}{\ln 2} D[2^x] = \frac{1}{\ln 2} 2^x \ln 2 = 2^x$

Ex. $D(x^2) = D(e^{2 \ln x}) = e^{2 \ln x} D(2 \ln x) = e^{2 \ln x}\left(\frac{2}{x}\right) = (x^2)\left(\frac{2}{x}\right) = 2x$

Ex. $D[\log_2 x] = D\left[\frac{\ln x}{\ln 2}\right] \overset{*}{=} \frac{1}{\ln 2} D[\ln x] \overset{*}{=} \frac{1}{\ln 2}\left(\frac{1}{x}\right)$ *basically the same as $D[\ln x]$, just scaled by $\frac{1}{\ln 2}$

Ex. $D[\log_3 x] = \frac{1}{x \ln 3}$

Ex. $D[x \log_3 x] = \log_3 x + x\left(\frac{1}{x \ln 3}\right) = \log_3 x + \frac{1}{\ln 3}$

Ex. $\int 10^x \, dx = \frac{10^x}{\ln 10} + C \longrightarrow D\left[\frac{10^x}{\ln 10} + C\right] = \frac{1}{\ln 10} D[10^x] + 0 = \frac{1}{\ln 10}\left(10^x \ln 10\right) = 10^x$

Exp and Log With Other Bases

Ex. $\int x\,10^{-x^2}\,dx = \int -\frac{1}{2}10^u\,du = -\frac{1}{2}\int 10^u\,du = -\frac{1}{2}\frac{1}{\ln 10}10^u + c$

$u = -x^2$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

$= \frac{-10^{-x^2}}{2\ln 10} + C$

Ex. $D\left[\log_3(\log_2(x))\right] = D\left[\log_3\left(\frac{\ln x}{\ln 2}\right)\right] = D\left[\log_3(\ln x) - \log_3(\ln 2)\right] = D\left[\frac{\ln(\ln x)}{\ln 3} - \log_3(\ln 2)\right]$

$= \frac{1}{\ln 3}D\left[\ln(\ln x)\right] - 0 = \frac{1}{\ln 3}\frac{1}{\ln x}D\left[\ln x\right] = \frac{1}{\ln 3}\frac{1}{\ln x}\frac{1}{x} = \left(\frac{1}{\ln 3}\right)\frac{1}{x\ln x}$