Optimization

Ex. 2 A cylyndrical can is to be made to hold 1 L of cola. Find the dimensions that will minimize production costs (minimize surface area).

$$V = 1\,L = \underbrace{1000\,cm^3 = \pi r^2 h}$$

$$h = \underbrace{\frac{1000}{\pi r^2}}$$

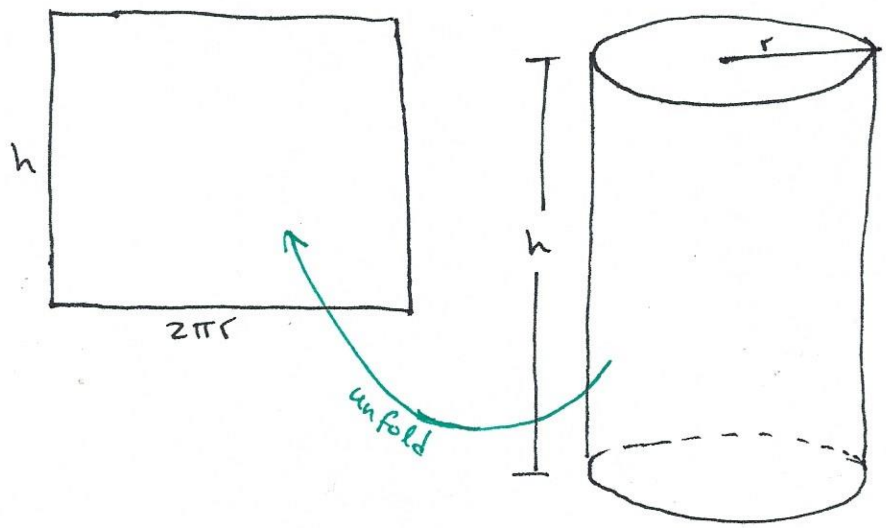
$r > 0$

$h > 0$

$$SA = \underbrace{2\pi r^2 + 2\pi r h}_{\text{minimize}}$$

$$= 2\pi r^2 + 2\pi r\left(\frac{1000}{\pi r^2}\right)$$

$$= 2\pi r^2 + \frac{2000}{r}$$



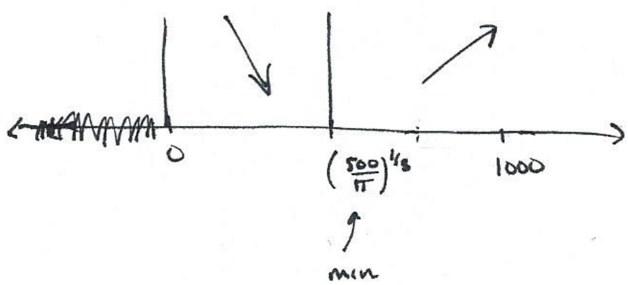
$$(SA)' = 4\pi r - \frac{2000}{r^2} = 0$$

$$4\pi r = \frac{2000}{r^2}$$

$$4\pi r^3 = 2000$$

$$r^3 = \frac{2000}{4\pi} = \frac{500}{\pi}$$

$$r = \left(\frac{500}{\pi}\right)^{1/3} \longleftarrow \text{crit \#}$$



$$h = \frac{1000}{\pi r^2} = \frac{2 \cdot 500}{\pi\left(\frac{500}{\pi}\right)^{2/3}} = \frac{2 \cdot 500}{\pi^{1/3} \cdot 500^{2/3}} = 2 \cdot \left(\frac{500}{\pi}\right)^{1/3} = 2r$$

$\rightarrow r = \left(\frac{500}{\pi}\right)^{1/3}$ cm

$\rightarrow h = 2r$ cm

Optimization

Math 2413
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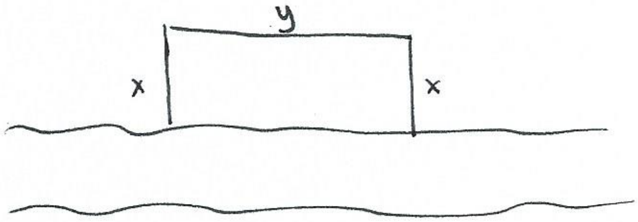
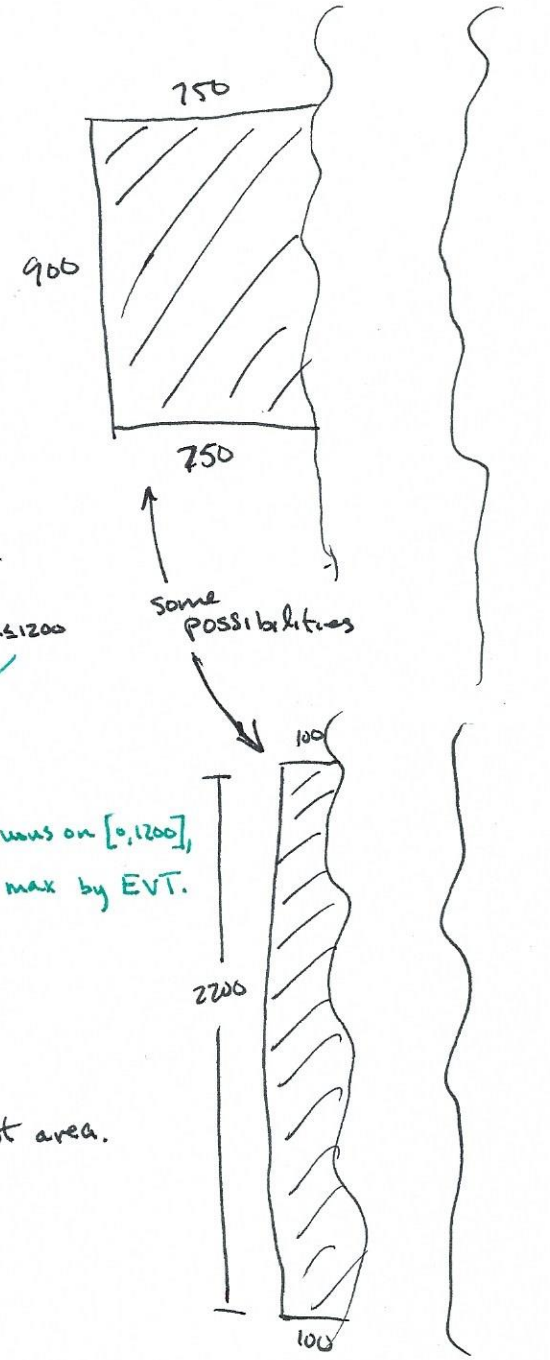
Ex. 1 A farmer has 2400 ft of fencing and wants to fence off a rectangular field that borders a straight river. (She needs no fence along the river). What are the dimensions of the field w/ the largest area?





$\underbrace{A = xy}_{\substack{\uparrow \\ \text{want to maximize}}}, \quad \underbrace{2x + y = 2400}_{\substack{\uparrow \\ \text{constraint}}}$

* negative distance doesn't make sense

$0 \le x \le 1200, \quad 0 \le y \le 2400$

$\Downarrow$

$y = 2400 - 2x \implies A = xy = x(2400 - 2x) = 2400x - 2x^2,$ $0 \le x \le 1200$

$A' = 2400 - 4x = 0$

$2400 = 4x$

one crit # $\longrightarrow$ $600 = x$

A is continuous on $[0, 1200]$, so A has max by EVT.

x	A(x)	
0	0	
600	720,000	$\longleftarrow$ max
1200	0	

So $x = 600$ ft, $y = 1200$ ft gives the largest area.