

Curve Sketching

Math 2413
Dr. Kennedy
6 Mar 2019
pg 40

Ex: $f(x) = x^{2/3} (6-x)^{1/3}$

y-int: $f(0) = 0$
x-int: $0, 6$

Domain: all reals \rightarrow no vertical asymptotes

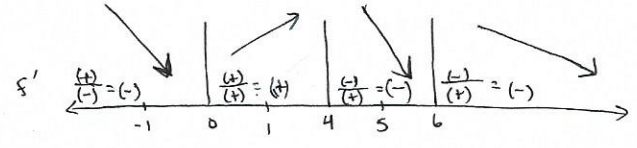
horizontal asymptotes? \rightarrow No.

$\lim_{x \rightarrow -\infty} x^{2/3} (6-x)^{1/3} = -\infty$
large (+) large (-)

$\lim_{x \rightarrow \infty} x^{2/3} (6-x)^{1/3} = +\infty$
large (+) large (+)

$f'(x) = \frac{4-x}{x^{4/3} (6-x)^{2/3}}$

\rightarrow crit #: $x = 4, 0, 6$



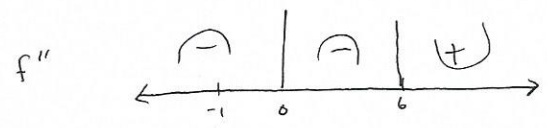
f dec on $(-\infty, 0) \cup (4, 6)$
 f inc on $(0, 4)$

$f(0) = 0 \rightarrow$ local min

$f(4) = 4^{2/3} (6-4)^{1/3} = (16)^{1/3} (2)^{1/3} = (8)^{1/3} (2)^{1/3} (2)^{1/3} = 2 \cdot 2^{1/3} \cdot 2^{1/3} = 2 \cdot 2^{2/3} = 2^{5/3} = 2^{4/3} \cdot 2^{1/3} = \frac{4}{2^{1/3}} \rightarrow$ local max

$f''(x) = \frac{-8}{x^{4/3} (6-x)^{5/3}}$

\rightarrow crit #: $x = 0, 6$



f concave down on $(-\infty, 0) \cup (0, 6)$
 f concave up on $(6, \infty)$

inflection point @ $(6, 0)$

