

Curve Sketching

Math 2413
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Pg 39

Ex. $f(x) = \frac{x^2}{1-x^2}$

Domain: $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$ \rightarrow vertical asymptotes @ $x = \pm 1$

$$\lim_{x \rightarrow 1^+} \frac{x^2}{1-x^2} = -\infty \quad \frac{+}{0^-}$$

$$\lim_{x \rightarrow 1^-} \frac{x^2}{1-x^2} = \infty$$

$$\lim_{x \rightarrow -1^+} \frac{x^2}{1-x^2} = \infty$$

$$\lim_{x \rightarrow -1^-} \frac{x^2}{1-x^2} = -\infty$$

vertical asymptotes

$$\lim_{x \rightarrow \infty} \frac{x^2}{1-x^2} = \lim_{x \rightarrow \infty} \frac{1}{\frac{1}{x^2} - 1} = -1$$

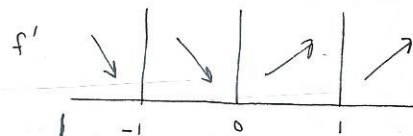
$$\lim_{x \rightarrow -\infty} \frac{x^2}{1-x^2} = \lim_{x \rightarrow -\infty} \frac{1}{\frac{1}{x^2} - 1} = -1$$

horizontal asymptotes

$$\begin{aligned} f'(x) &= [x^2(1-x^2)^{-1}]' = 2x(1-x^2)^{-1} + (-1)x^2(1-x^2)^{-2}(-2x) \\ &= 2x \left[\frac{1}{1-x^2} + \frac{x^2}{(1-x^2)^2} \right] \\ &= 2x \left[\frac{1-x^2+x^2}{(1-x^2)^2} \right] = \frac{2x}{(1-x^2)^2} \end{aligned}$$

f' undefined at $x = \pm 1$

$f' = 0$ when $2x = 0$
 $x = 0$

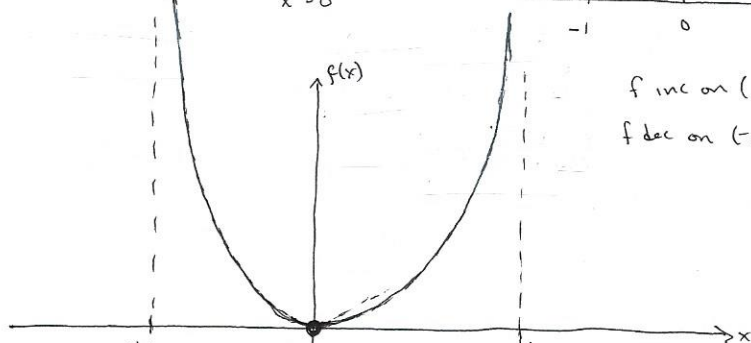


$$f''(x) = \frac{2(1-x^2)^2 - (2x)(2)(1-x^2)(-2x)}{(1-x^2)^4}$$

$$= \frac{2(1-x^2)(1-x^2+4x^2)}{(1-x^2)^4} = \frac{2(1+3x^2)}{(1-x^2)^3}$$

f'' not defined at $x = \pm 1$

$f'' \neq 0$ because $1+3x^2 > 0$



f inc on $(0, 1) \cup (1, \infty)$

f dec on $(-\infty, -1) \cup (-1, 0)$

