Math 2413
Dr. Kennedy
28 Mar 2019
pg S1

Definite Integrals

Properties of definite integrals

① $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

② $\int_a^a f(x)\,dx = 0$

③ $\int_a^b c\,dx = c(b-a)$

④ $\int_a^b c f(x)\,dx = c\int_a^b f(x)\,dx$

⑤ $\int_a^b [f(x) \pm g(x)]\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

⑥ $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$ (for f defined on interval containing a, b, c)

⑦ if $f(x) \geq 0$ for $x \in [a,b]$, then $\int_a^b f(x)\,dx \geq 0$.

⑧ if $f(x) \geq g(x)$ for $x \in [a,b]$, then $\int_a^b f(x)\,dx \geq \int_a^b g(x)\,dx$

⑨ if $m \leq f(x) \leq M$ for $x \in [a,b]$, then $m(b-a) \leq \int_a^b f(x)\,dx \leq M(b-a)$

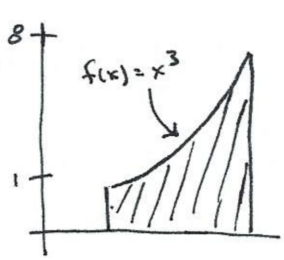
FUNDAMENTAL THEOREM OF CALCULUS (I)

Thm. If f is continuous on the closed interval $[a,b]$, then

$$\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)$$

where F is any antiderivative of f on $[a,b]$.

Ex.

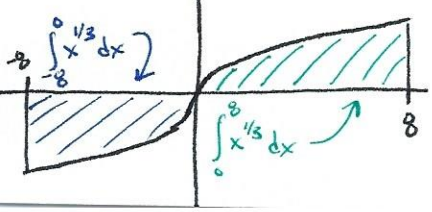


antiderivative of x^3 : $\frac{x^4}{4} = F(x)$

$\int_1^2 f(x)\,dx = F(x)\Big|_1^2 = F(2) - F(1)$

$\int_1^2 x^3\,dx = \frac{x^4}{4}\Big|_1^2 = \frac{(2)^4}{4} - \frac{(1)^4}{4} = \frac{15}{4}$ ← exact area

Ex. $\int_{-8}^{8} x^{1/3}\,dx = \frac{x^{4/3}}{4/3}\Big|_{-8}^{8} = \frac{3}{4}x^{4/3}\Big|_{-8}^{8} = \frac{3}{4}\left[(8)^{4/3} - (-8)^{4/3}\right] = \frac{3}{4}\left[(8)^{4/3} - (8)^{4/3}\right] = 0$



$(-8)^{4/3} = \left[(-8)^{1/3}\right]^4 = \left[(-2)\right]^4 = 16$

$(8)^{4/3} = \left[(8)^{1/3}\right]^4 = \left[(2)\right]^4 = 16$

Definite Integrals

Math 2413
Dr. Kennedy
28 Mar 2019
pg 52

Ex. $\int_1^2 (x-2)\,dx = \left(\frac{x^2}{2} - 2x\right)\Big|_1^2 = \left(\frac{2^2}{2} - 2(2)\right) - \left(\frac{1^2}{2} - 2(1)\right)$

$= (2-4) - (\frac{1}{2} - 2)$

$= -\frac{1}{2}$

Ex. $\int_{-3}^1 8\,dt = 8t\Big|_{-3}^1 = 8(1--3) = 32$

Ex. $\int_0^2 (2+\cos x)\,dx = \int_0^2 2\,dx + \int_0^2 \cos x\,dx = 2x\Big|_0^2 + \sin x\Big|_0^2 = 2(2-0) + (\sin(2) - \sin(0))$

$= 4 + \sin(2)$

Ex. $\int_1^8 \sqrt{2/x}\,dx = \int_1^8 \left(\frac{2}{x}\right)^{1/2} dx = \int_1^8 \frac{2^{1/2}}{x^{1/2}}\,dx = 2^{1/2}\int_1^8 x^{-1/2}\,dx = 2^{1/2}\frac{x^{1/2}}{1/2}\Big|_1^8 = 2^{3/2}\left(8^{1/2} - 1^{1/2}\right)$

$= 8^{1/2}(8^{1/2} - 1) = 8 - 8^{1/2} = 8 - 2\sqrt{2}$