Definite Integrals

Math 2413
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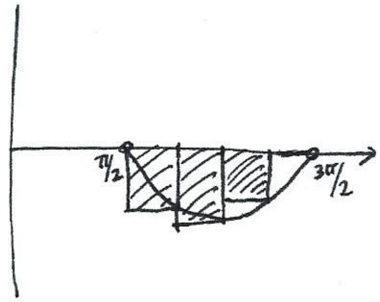
27 Mar 2019

Ex. $f(x) = \cos(x)$, $x \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$



$n = 4 \rightarrow \Delta x = \frac{b-a}{4} = \frac{\pi}{4}$

$x_0 = \pi/2$, $x_1 = \frac{3\pi}{4}$, $x_2 = \pi$, $x_3 = \frac{5\pi}{4}$, $x_4 = \frac{3\pi}{2}$

$\bar{x}_i = x_i$

$$\sum_{k=1}^{4} f(\bar{x}_k)\Delta x = f\left(\frac{3\pi}{4}\right)\left(\frac{\pi}{4}\right) + f(\pi)\left(\frac{\pi}{4}\right) + f\left(\frac{5\pi}{4}\right)\left(\frac{\pi}{4}\right) + f\left(\frac{3\pi}{2}\right)\left(\frac{\pi}{4}\right)$$

$$= \frac{\pi}{4}\left[\cos\left(\frac{3\pi}{4}\right) + \cos(\pi) + \cos\left(\frac{5\pi}{4}\right) + \cos\left(\frac{3\pi}{2}\right)\right]$$

$$= \frac{\pi}{4}\left[-\frac{1}{\sqrt{2}} + (-1) + \left(\frac{-1}{\sqrt{2}}\right) + 0\right]$$

$$= -\frac{\pi}{4}\left[\frac{2}{\sqrt{2}} + 1\right] = -\frac{\pi}{4}\left[\sqrt{2} + 1\right]$$

Properties of Definite Integrals (from limit laws and algebra properties)

① $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

② $\int_a^a f(x)\,dx = 0$

③ $\int_a^b c\,dx = c(b-a)$

④ $\int_a^b [f(x) \pm g(x)]\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

⑤ $\int_a^b c f(x)\,dx = c\int_a^b f(x)\,dx$

⑥ If f is integrable on an interval containing a, b, c, then $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

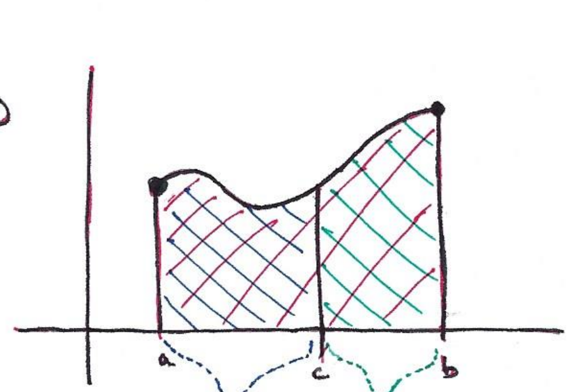
④ $\int_a^b [f(x) \pm g(x)]\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} [f(\bar{x}_i) \pm g(\bar{x}_i)]\Delta x$

$= \lim_{n\to\infty} \sum_{i=1}^{n} [f(\bar{x}_i)\Delta x \pm g(\bar{x}_i)\Delta x]$

$= \lim_{n\to\infty} \sum_{i=1}^{n} f(\bar{x}_i)\Delta x \pm \lim_{n\to\infty} \sum_{i=1}^{n} g(\bar{x}_i)\Delta x$

$= \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

⑥



$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$