Antiderivatives

Math 2413
Dr. Kennedy
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Pg 46

Ex. Suppose $f'(x) = 2x - 3x^{-4}$, $x>0$, $f(1)=3$. Find f.

$$f(x) = x^2 - 3\frac{x^{-4+1}}{-4+1} + C$$

$$= x^2 + x^{-3} + C$$

$$f(1) = 1^2 + 1^{-3} + C = 3 \implies f(x) = x^2 + x^{-3} + 1$$

$$1 + 1 + C = 3$$

$$C = 1$$

Ex. Find f for $f''(\theta) = \sin\theta + \cos\theta$, $f(0)=3$, $f'(0)=4$.

$$f'(\theta) = -\cos\theta + \sin\theta + C$$

$$f(\theta) = -\sin\theta - \cos\theta + C\theta + D$$

$$f(0) = -\sin(0) - \cos(0) + C\cdot 0 + D = 3$$

$$-1 + D = 3$$

$$D = 4$$

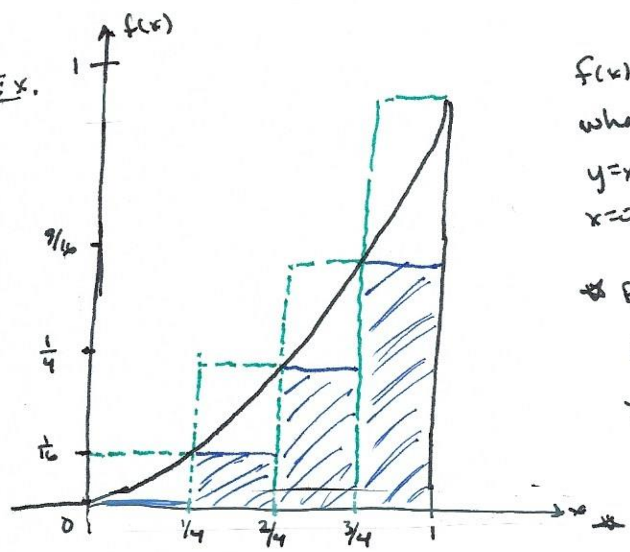
$$f'(0) = -\cos(0) + \sin(0) + C = 4$$

$$-1 + C = 4$$

$$C = 5$$

$$\implies f(\theta) = -\sin\theta - \cos\theta + 5\theta + 4$$

Ex.



$f(x) = x^2$

what is the are of the region bounded by the curves

$y = x^2$, $y = 0$

$x = 0$, $x = 1$

* Partition $[0,1]$ into 4 equal length subintervals. Each subinterval has length $1/4$.

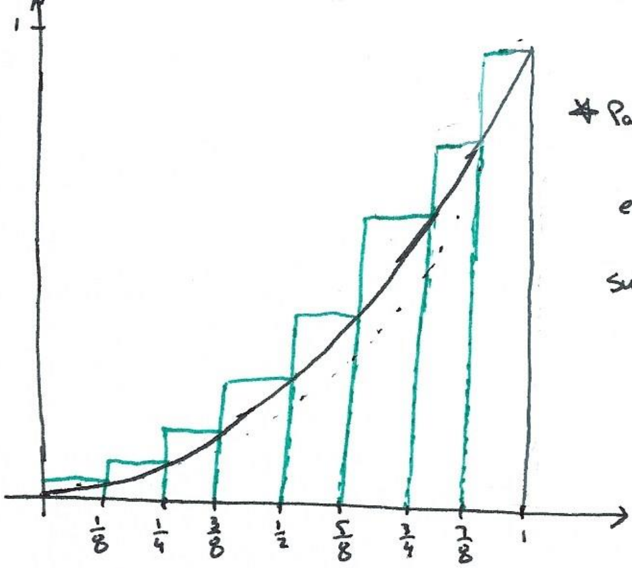
  The endpoints are $\{0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1\}$

est. of area (upper): $\frac{1}{4}f(\frac{1}{4}) + \frac{1}{4}f(\frac{1}{2}) + \frac{1}{4}f(\frac{3}{4}) + \frac{1}{4}f(1) = \frac{1}{4}\left[\frac{1}{16} + \frac{1}{4} + \frac{9}{16} + 1\right] = \frac{15}{32}$ (over-estimate)

est. of area (lower): $\frac{1}{4}(0)^2 + \frac{1}{4}(\frac{1}{4})^2 + \frac{1}{4}(\frac{1}{2})^2 + \frac{1}{4}(\frac{3}{4})^2 = \frac{1}{4}\left[\frac{1}{16} + \frac{1}{4} + \frac{9}{16}\right] = \frac{1}{4}\cdot\frac{14}{16} = \frac{7}{32}$ (under-estimate)

$$\implies \frac{7}{32} < \text{area} < \frac{15}{32}$$

Ex (cont.)



* Partition $[0,1]$ into 8 equal subintervals

endpoints: $\{0, \frac{1}{8}, \frac{1}{4}, \frac{3}{8}, \frac{1}{2}, \frac{5}{8}, \frac{3}{4}, \frac{7}{8}, 1\}$

subint length: $\frac{1}{8}$

upper est: $\frac{1}{8} \cdot \frac{1}{64}\left[1+4+9+16+25+36+49+64\right] = \frac{1}{8}\frac{1}{64}(204) = \frac{51}{128}$

lower est: $\frac{1}{8} \cdot \frac{1}{64}\left[0+1+4+9+16+25+36+49\right] = \frac{1}{8}\frac{1}{64}(140) = \frac{35}{128}$

$\Rightarrow \frac{35}{128} < \text{area} < \frac{51}{128}$