* Re-Scan for 3122 or 3123 *

Example 4.5.10. cont

(g) $\lim_{x \to \infty} \frac{x - \cos x}{x}$

$\lim_{x \to \infty} \frac{x}{x} - \lim_{x \to \infty} \frac{\cos x}{x}$

$\lim_{x \to \infty} 1 - \lim_{x \to \infty} \frac{\cos x}{x}$

$1 - \lim_{x \to \infty} \frac{\cos x}{x}$

$1 - 0 = \boxed{1}$

cos x oscillates between -1 and 1 as $x \to \infty$.

$\frac{\cos x}{x} = 0$ since the denominator grows faster as $x \to \infty$

How to show

Recall the Squeeze Theorem

find $\lim_{x \to c} f(x)$

If $g(x) \leq f(x) \leq h(x)$ and

$\lim_{x \to c} g(x) = \lim_{x \to c} h(x) = L$,

then $\lim_{x \to c} f(x) = L$

we know $-1 \leq \cos x \leq 1$,

so $-\frac{1}{x} \leq \frac{\cos x}{x} \leq \frac{1}{x}$. ← this is our $g(x) \leq f(x) \leq h(x)$

$\lim_{x \to \infty} -\frac{1}{x} = 0$ and $\lim_{x \to \infty} \frac{1}{x} = 0$ ← $\lim_{x \to c} g(x) = \lim_{x \to c} h(x) = 0$

$\lim_{x \to \infty} -\frac{1}{x} = \lim_{x \to \infty} \frac{1}{x} = 0$

so $\lim_{x \to c} f(x) = 0$

and $-\frac{1}{x} \leq \frac{\cos x}{x} \leq \frac{1}{x}$.

Therefore by the Squeeze Theorem,

$\lim_{x \to \infty} \frac{\cos x}{x} = 0$.