Antiderivatives

If $f'(x) = \sin(x)$, what is f?

$D[\sin(x)] = \cos x$ $\quad f(x) = -\cos x$ $\quad g(x) = 2 - \cos x$

$D[\cos x] = -\sin x$ $\quad f'(x) = -(-\sin x) = \sin x$ $\quad g'(x) = 0 + \sin x = \sin x = f'(x)$

$\Rightarrow f(x) = -\cos x + C$, where C is any constant

$f'(x) = x \Rightarrow f(x) = \frac{x^2}{2}$

* $D[x^n] = nx^{n-1}$

It looks like if $f'(x) = x^n$, then $f(x) = \frac{x^{n+1}}{n+1}$.

Check: $f(x) = \frac{x^{n+1}}{n+1}$

Then $f'(x) = \frac{n+1}{n+1} x^{n+1-1} = x^n$ ✓

function	antiderivative
$\sin x$	$-\cos x + C$
$\cos x$	$\sin x + C$
x	$\frac{x^2}{2} + C$
x^n, $n \neq -1$	$\frac{x^{n+1}}{n+1} + C$
$cf(x)$	$cF(x) + C$
1	$x + C$
$f(x) + g(x)$	$F(x) + G(x) + C$
$\sec^2 x$	$\tan x + C$

Ex. $f(x) = x - 3$

Guess: $F(x) = \frac{x^2}{2} - 3x + C$

Check: is $F'(x) = f(x)$

$F'(x) = \frac{1}{2}(2x) - 3 + 0$

$= x - 3$ ✓

Ex antiderivative for $f(x) + g(x)$ is $F(x) + G(x) + C$

Ex. is the product of antiderivatives the antiderivative of $f(x)$?

$f(x) = (\sin x)(x^2 + 2)$

$F(x) \stackrel{?}{=} (-\cos x)(\frac{x^3}{3} + 2x) + C$

$F'(x) = (\sin x)(\frac{x^3}{3} + 2x) + (-\cos x)(x^2 + 2) + 0$

$\neq f(x)$

more:

function	$\frac{1}{1+x^2}$	$\frac{1}{\sqrt{1-x^2}}$	$\frac{1}{\lvert x \rvert \sqrt{x^2-1}}$
antiderivative	$\arctan x + C$	$\arcsin x + C$	$\operatorname{arcsec} x + C$