Math 2413
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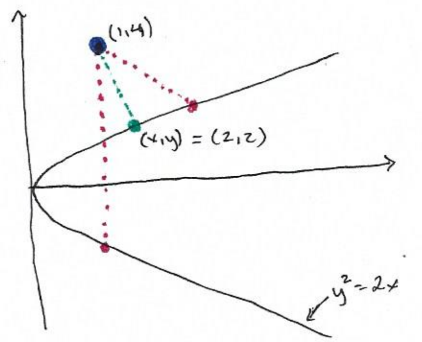
Optimization

Ex. Find the point on the graph of $y^2=2x$ that is closest to the point $(1,4)$.



distance: $d = \left[(x-1)^2+(y-4)^2\right]^{1/2} = \left[\left(\frac{y^2}{2}-1\right)^2+(y-4)^2\right]^{1/2}$ * domain (d) = $\mathbb{R}$

$y^2=2x \Rightarrow x=\frac{y^2}{2}$

Now we have a function $g(y)=\left[\left(\frac{y^2}{2}-1\right)^2+(y-4)^2\right]^{1/2}$ of a single variable.

Minimize $g(y)$: $g'(y)=\frac{1}{2}\left[\left(\frac{y^2}{2}-1\right)^2+(y-4)^2\right]^{-1/2}\left[2\left(\frac{y^2}{2}-1\right)\left(\frac{2y}{2}\right)+2(y-4)(1)\right]$

$= \dfrac{2\left(\frac{y^2}{2}-1\right)y+2(y-4)}{2\left[\left(\frac{y^2}{2}-1\right)^2+(y-4)^2\right]^{1/2}} = \dfrac{\frac{1}{2}y^3-y+y-4}{\left[\left(\frac{y^2}{2}-1\right)^2+(y-4)^2\right]^{1/2}} = \dfrac{\frac{1}{2}y^3-4}{\left[\left(\frac{y^2}{2}-1\right)^2+(y-4)^2\right]^{1/2}}$

$g'(y)=0 \Rightarrow \frac{1}{2}y^3=4$

$y^3=8$

☆ $y=2$

* note that the denominator of g' is always positive since it represents the distance from the parabola to a point not on the parabola.

g': ↘ | ↗ on the number line with 0, 2, 10

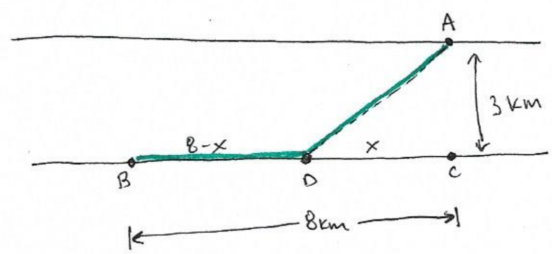
$g(2)$ is a local and absolute min

$g(2)=\sqrt{5}$ is the min. distance.

☆ $y=2 \Rightarrow x=\frac{(2)^2}{2}=2$

$\Longrightarrow$ the point on the parabola closest to $(1,4)$ is $(2,2)$

Ex.



A man launches his boat from point A on the bank of a straight river, 3 km wide, and wants to reach point B, 8 km downriver, on the opposite bank. He will use a combination of rowing (6 km/hr) and running (8 km/hr). Where should he land to reach B as soon as possible?

$\longrightarrow$ minimize time $T(x)$

distance rowing $=(x^2+9)^{1/2} \Rightarrow$ time rowing $=\frac{(x^2+9)^{1/2}}{6}$

distance running $=8-x \Rightarrow$ time running $=\frac{8-x}{8}=1-\frac{x}{8}$

$x=0$ means he rowed straight across 3km then ran 8km

$x=8$ means he rowed straight to B, with no running

$T(x)=$ (time rowing) + (time running)

$=\frac{(x^2+9)^{1/2}}{6}+1-\frac{x}{8}$, domain $(T)=[0,8]$

$T'(x)=\frac{1}{6}\cdot\frac{1}{2}(x^2+9)^{-1/2}(2x)-\frac{1}{8}$

$=\frac{x}{6(x^2+9)^{1/2}}-\frac{1}{8}=0$

$\frac{x}{6(x^2+9)^{1/2}}=\frac{1}{8}$

$8x=6(x^2+9)^{1/2}$

$64x^2=36(x^2+9)$

$28x^2=36\cdot 9$

$x^2=\frac{36\cdot 9}{28}=\frac{9\cdot 9}{7}$

$x=\frac{9}{\sqrt{7}}$

x	$T(x)$
0	$\frac{3}{6}+1=\frac{3}{2}$
8	$\frac{\sqrt{73}}{6}$; $\frac{8}{6}<\frac{\sqrt{73}}{6}<\frac{9}{6}$; ≈ 1.424; $\frac{4}{3}<\frac{\sqrt{73}}{6}<3$
$\frac{9}{\sqrt{7}}$	$\frac{\left(\frac{81}{7}+9\right)^{1/2}}{6}+1-\frac{9}{8\sqrt{7}}$