Ex. Find the point on the graph of \( y^2 = 2x \) that is closest to the point \((1,4)\).

Distance: \[
d = \sqrt{(x-1)^2 + (y-4)^2} = \sqrt{\left(\frac{y^2}{2} - 1\right)^2 + (y-4)^2} \]

* Domain \( (d) = \mathbb{R} \)

\( y^2 = 2x \Rightarrow x = \frac{y^2}{2} \)

Now we have a function \( g(y) = \left(\frac{y^2}{2} - 1\right)^2 + (y-4)^2 \) of a single variable.

Minimize \( g(y) \):

\[
g'(y) = \frac{1}{2} \left(\frac{y^2}{2} - 1\right) + 2(y-4) = \frac{y^2}{4} - \frac{1}{2} + 2y - 8 = \frac{y^2 - 8}{2} \]

\( g'(y) = 0 \Rightarrow \frac{y^2}{4} = 8 \)

\( y = \pm 2 \)

* Note that the denominator of \( g' \) is always positive, so since it represents the distance from the parabola to a point not on the parabola.

\( \therefore y = 2 \Rightarrow x = \frac{2^2}{2} = 2 \)

\( g(2) \) is a local and absolute min.

\( g(2) = 5^2 \) is the minimum distance.

\[ \Rightarrow \text{the point on the parabola closest to } (1,4) \text{ is } (\frac{1}{2}, 2) \]

A man knows his boat from point A on the bank of a straight river, 3 km wide, and wants to reach point B, 8 km downstream, on the opposite bank. He will use a combination of rowing (6 km/hr) and running (8 km/hr). Where should he land to reach B as soon as possible?

- \( T(x) = \) time rowing + time running

\[
T(x) = \frac{(x^2 + 9)^{1/2}}{6} + 1 - \frac{x}{8}, \quad \text{domain } (T) = [0, 8]
\]

\[
T'(x) = \frac{x}{6(x^2 + 9)^{1/2}} - \frac{1}{8}
\]

\( x = \frac{1}{6} \) means he rowed straight across 3 km then ran 8 km.

\( x = 8 \) means he rowed straight to B, with no running.

\[
\text{distance rowing} = (x^2 + 9)^{1/2} \Rightarrow \text{time rowing} = \frac{(x^2 + 9)^{1/2}}{6}
\]

\[
\text{distance running} = 8-x \Rightarrow \text{time running} = \frac{8-x}{8}
\]

\( x = 0 \) means he rowed straight across 3 km then ran 8 km.

\( x = 8 \) means he rowed straight to B, with no running.