Differentiation Techniques

Chain Rule

Suppose g is differentiable at x, and f is differentiable at $g(x)$.

Then $F = f \circ g$ defined by $F(x) = f(g(x))$ is differentiable at x and

$$F'(x) = f'(g(x))g'(x)$$

Ex. $F(x) = \sqrt{x^2+1}$

outside: $f(u) = \sqrt{u}$ $\quad f'(u) = \frac{1}{2}u^{-1/2}$

inside: $g(x) = x^2+1 = u$ $\quad g'(x) = 2x$

$$F'(x) = f'(g(x))g'(x) = f'(u)g'(x) = \left(\tfrac{1}{2}u^{-1/2}\right)(2x) = \tfrac{1}{2}\cdot 2\cdot (x^2+1)^{-1/2}x = \frac{x}{(x^2+1)^{1/2}} = \frac{x}{\sqrt{x^2+1}}$$

Ex. $y = (x^3-1)^{100}$

outside $f(u) = u^{100}$ $\quad f'(u) = 100u^{99}$

$g(x) = x^3-1 = u$ $\quad g'(x) = 3x^2$

$$y' = f'(u)g'(x) = (100u^{99})(3x^2) = 300x^2(x^3-1)^{99}$$

Ex. $y = \sin(x^{10})$

outside: $f(u) = \sin(u)$

inside: $g(x) = x^{10}$

$f'(u) = \cos(u)$

$g'(x) = 10x^9$

$y' = f'(u)g'(x) = \cos(u)(10x^9)$

$= 10x^9\cos(x^{10})$

$y = \sin^{10}(x) = [\sin(x)]^{10}$

outside: $f(u) = u^{10}$

inside: $g(x) = \sin(x)$

$f'(u) = 10u^9$

$g'(x) = \cos(x)$

$y' = f'(u)g'(x) = (10u^9)\cos(x)$

$= 10\sin^9(x)\cos(x)$

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pg 20

Chain Rule

Ex. $y = \sin\left[\cos\left(\tan(x)\right)\right]$

outside: $f(z) = \sin(z)$

inside: $g(x) = \cos\left(\tan(x)\right) = z$

$f'(z) = \cos(z)$

$g'(x) = \left[-\sin\left(\tan(x)\right)\right]\left[\sec^2(x)\right]$

$z = \cos\left[\tan(x)\right]$

outside: $f(u) = \cos(u)$, $f'(u) = -\sin(u)$

inside: $g(x) = \tan(x) = u$, $g'(x) = \sec^2(x)$

$z' = f'(u)g'(x) = -\sin(u)\sec^2(x)$

$= \left[\sin\left(\tan(x)\right)\right]\left[\sec^2(x)\right]$

$y' = f'(z)g'(x)$

$= \left(\cos\left[\cos\left(\tan(x)\right)\right]\right)\left(-\sin\left(\tan(x)\right)\sec^2(x)\right)$

Ex. $y = \frac{1}{\sqrt[3]{4x+1}} = (4x+1)^{-1/3}$

outside: $f(u) = u^{-1/3}$, $f'(u) = -\frac{1}{3}u^{-4/3}$

inside: $g(x) = 4x+1 = u$, $g'(x) = 4x$

$y' = f'(u)g'(x) = \left(-\frac{1}{3}u^{-4/3}\right)(4x)$

$= -\frac{1}{3}(4x+1)^{-4/3}(4x) = -\frac{4}{3}x(4x+1)^{-4/3}$

Ex. $y = \frac{1}{\sqrt{x^2+x+1}} = (x^2+x+1)^{-1/2}$

outside: $f(u) = u^{-1/2}$, $f'(u) = -\frac{1}{2}u^{-3/2}$

inside: $g(x) = x^2+x+1$, $g'(x) = 2x+1$

$y' = f'(u)g'(x) = \left(-\frac{1}{2}u^{-3/2}\right)(2x+1)$

$= -\frac{1}{2}(x^2+x+1)^{-3/2}(2x+1)$