Differentiation Techniques

Math 2413
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Recall:

1) $D[c] = 0$
2) $D[x^n] = nx^{n-1}$, $n \in \mathbb{R}$, x^n defined.
3) $D[cf(x)] = c\,D[f(x)]$, c a constant
4) $D[f(x) \pm g(x)] = [D(f(x))] \pm [D(g(x))]$
5) $D[f(x)g(x)] = [D(f(x))]g(x) + f(x)[D(f(x))]$
6) $D\left[\frac{f(x)}{g(x)}\right] = \dfrac{D[f(x)]g(x) - f(x)[D(f(x))]}{(g(x))^2}$

Find derivative of $\sin(x)$. Assume $\lim_{h\to 0} \frac{\sin(h)}{h} = 1$

$$D[\sin(x)] = \lim_{h\to 0} \frac{\sin(x+h) - \sin(x)}{h} \overset{*}{=} \lim_{h\to 0} \frac{1}{h}\left[\sin(x)\cos(h) - \sin(h)\cos(x) - \sin(x)\right]$$

$*\ \sin(x \pm y) = \sin(x)\cos(y) \pm \sin(y)\cos(x)$

$*\ \cos^2(x) + \sin^2(x) = 1$

$$= \lim_{h\to 0}\left[\frac{\sin(x)\cos(h) - \sin(x)}{h}\right] + \lim_{h\to 0}\left[\frac{-\sin(h)\cos(x)}{h}\right]$$

$\frac{*}{*}$ x is a constant here since h is the one changing

$$= \sin(x) \lim_{h\to 0}\left[\frac{\cos(h) - 1}{h}\right] + \cos(x)\lim_{h\to 0}\left[-\frac{\sin(h)}{h}\right]$$

$$= \sin(x)[0] + \cos(x)[1] = \boxed{\cos(x)} = D[\sin(x)]$$

$***$

$$\lim_{h\to 0}\left(\frac{\cos(h) - 1}{h}\right)\left(\frac{\cos(h)+1}{\cos(h)+1}\right) = \lim_{h\to 0} \frac{\cos^2(h) - 1}{h(\cos(h)+1)} = \lim_{h\to 0} \frac{\sin^2(h)}{h(\cos(h)+1)}$$

$$= \lim_{h\to 0}\left(\frac{\sin(h)}{h}\right)\left(\frac{-\sin(h)}{\cos(h)+1}\right) = \left[\lim_{h\to 0}\frac{\sin(h)}{h}\right]\cdot\left[\lim_{h\to 0}\frac{-\sin(h)}{\cos(h)+1}\right]$$

$$= (1)\left(\tfrac{0}{2}\right) = 0$$

$$\Longrightarrow \frac{d}{dx}[\sin(x)] = \cos(x)$$

$$\frac{d}{dx}[\cos(x)] = -\sin(x)$$

$$D[\tan(x)] = D\left[\frac{\sin(x)}{\cos(x)}\right] = \frac{D[\sin(x)]\cos(x) - \sin(x)D[\cos(x)]}{\cos^2(x)} = \frac{\cos(x)\cos(x) - \sin(x)(-\sin(x))}{\cos^2(x)}$$

$$= \frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)} = \frac{1}{\cos^2(x)} = \sec^2(x)$$

$$D[\cot(x)] = D\left[\frac{\cos(x)}{\sin(x)}\right] = \ldots = -\csc^2(x)$$

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$$D[\sec(x)] = D\left[\frac{1}{\cos(x)}\right] = \frac{0\cdot\cos(x) - 1\cdot(-\sin(x))}{\cos^2(x)} = \frac{\sin(x)}{\cos^2(x)} = \frac{\sin(x)}{\cos(x)}\cdot\frac{1}{\cos(x)}$$

$$= \tan(x)\sec(x)$$

$$D[\csc(x)] = \ldots = -\csc(x)\cot(x)$$

Ex. Find $f'(x)$:

$$f(x) = \frac{\sec(x)}{1+\tan(x)}$$

$$f'(x) = \frac{D[\sec(x)](1+\tan(x)) - \sec(x)\,D[1+\tan(x)]}{(1+\tan(x))^2}$$

$$= \frac{\sec(x)\tan(x)(1+\tan(x)) - \sec(x)(0+\sec^2(x))}{(1+\tan(x))^2} = \frac{\sec(x)(1+\tan(x)) - \sec^3(x)}{(1+\tan(x))^2}$$

$$f(x) = \cos(x)\cot(x)$$

$$f'(x) = D[\cos(x)]\cot(x) + \cos(x)\,D[\cot(x)]$$

$$= -\sin(x)\cot(x) + \cos(x)(-\csc^2(x))$$