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Cont ex 2

Use the definition of slope to find the slope of the tangent line for $g(x) = x^2+1$, points $(0,1)$, $(-1,2)$ and

$$m = \lim_{\Delta x \to 0} \frac{g(c+\Delta x) - g(c)}{\Delta x}$$

- for $(-1,2)$, $c=-1$

$$m = \lim_{\Delta x \to 0} \frac{g(-1+\Delta x) - g(-1)}{\Delta x}$$ → substitute $c=-1$

$$= \lim_{\Delta x \to 0} \frac{(\Delta x - 1)(\Delta x - 1) + 1 - 2}{\Delta x}$$ → substitute $g(-1+\Delta x) = (\Delta x - 1)^2 + 1$, $g(-1) = 2$

$$= \lim_{\Delta x \to 0} \frac{\Delta x^2 - 2\Delta x + \cancel{1+1-2}}{\Delta x}$$ → simplify $1+1-2=0$

$$= \lim_{\Delta x \to 0} \frac{\Delta x^2 - 2\Delta x}{\Delta x} = \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}(\Delta x - 2)}{\cancel{\Delta x}}$$ → factor out Δx and simplify $\frac{\Delta x}{\Delta x} = 1$

$$= \lim_{\Delta x \to 0} \Delta x - 2 = \lim_{\Delta x \to 0} \Delta x - \lim_{\Delta x \to 0} 2$$ → split limits and take limits individually

$$= 0 - 2 = -2. \quad \boxed{m = -2 \text{ for } (-1,2)}$$

Def. 3.1.3 If f is continuous at c and

$$\lim_{\Delta x \to 0} \frac{f(c+\Delta x) - f(c)}{\Delta x} = \infty \text{ or } \lim_{\Delta x \to 0} \frac{f(c+\Delta x) - f(c)}{\Delta x} = -\infty$$

then the vertical line at $x=c$ passing through $(c, f(c))$ is a vertical tangent line to the graph f.