Math 2413
Dr. Kennedy
4 Feb 2019
pg 14

Derivatives

$y = f(x)$

Notation: $y' = f'(x) = \frac{dy}{dx} = \frac{d(f(x))}{dx} = \frac{d}{dx} f(x) = D f(x)$

Techniques of Differention:

1) $\frac{d}{dx}[c] = 0$
2) $\frac{d}{dx}[x^n] = nx^{n-1}$, $n \in \mathbb{R}$, x^n defined
3) $\frac{d}{dx}[cf(x)] = c\left[\frac{d}{dx} f(x)\right]$
4) $\frac{d}{dx}[f(x) \pm g(x)] = \left[\frac{d}{dx} f(x)\right] \pm \left[\frac{d}{dx} g(x)\right]$
5) $\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$
6) $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$

$$\begin{cases} f(x) = c \\ f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{x \to a} \frac{c - c}{x - a} = \lim_{x \to a} \frac{0}{x - a} = 0 \end{cases}$$

$$\begin{cases} f(x) = x^1 \\ f'(a) = \lim_{x \to a} \frac{\overset{f(x)}{x} - \overset{f(a)}{a}}{x - a} = \lim_{x \to a} 1 = 1 = 1 \cdot a^0 \end{cases}$$

$$\begin{cases} f(x) = x^2 \\ f'(a) = \lim_{x \to a} \frac{x^2 - a^2}{x - a} = \lim_{x \to a} \frac{(x-a)(x+a)}{x-a} = \lim_{x \to a} (x+a) = 2a \\ f'(x) = 2x^{2-1} \end{cases}$$

$$\begin{cases} f(x) = x^n, \ n \text{ a pos. int} \\ f'(a) = \lim_{x \to a} \frac{x^n - a^n}{x - a} = \lim_{x \to a} \frac{x-a}{x-a}\left(x^{n-1} + x^{n-2}a + x^{n-3}a^2 + \dots + xa^{n-2} + a^{n-1}\right) \\ = \lim_{x \to a}\left[x^{n-1} + x^{n-2}a + \dots + xa^{n-2} + a^{n-1}\right] = a^{n-1} + a^{n-2}a + \dots + aa^{n-2} + a^{n-1} = na^{n-1} \end{cases}$$

$$\begin{cases} f(x) = \frac{1}{x} = x^{-1} \quad (n = -1) \\ f'(a) = \lim_{x \to a} \frac{\frac{1}{x} - \frac{1}{a}}{x - a} = \lim_{x \to a} \frac{\frac{a-x}{ax}}{x-a} = \lim_{x \to a} \frac{a-x}{(x-a)ax} = \lim_{x \to a} \frac{-1}{ax} = \frac{-1}{a^2} = -1 \cdot a^{-2} = -1 \cdot a^{-1-1} \\ f'(x) = \frac{-1}{x^2} = -1 \cdot x^{-1-1} \end{cases}$$

Derivative Techniques

Math 2413
Dr. Kennedy
4 Feb 2019
pg 15

Ex. Differentiate using the techniques

$\frac{1}{x^n} = x^{-n}$

$\sqrt[n]{x} = x^{\frac{1}{n}}$

$f(x) = x^6$	$f'(x) = 6x^5$
$f(x) = \pi$	$f'(x) = 0$
$f(x) = x^{1/2}$	$f'(x) = \frac{1}{2}x^{-1/2}$
$y = \sqrt[3]{x} = x^{1/3}$	$y' = \frac{1}{3}x^{-2/3}$
$y = \frac{1}{x^4} = x^{-4}$	$y' = -4x^{-5}$

$$y(x) = cf(x)$$

$$y'(a) = \lim_{x\to a} \frac{cf(x) - cf(a)}{x-a} = \lim_{x\to a} c\left[\frac{f(x)-f(a)}{x-a}\right] = c\left[\lim_{x\to a}\frac{f(x)-f(a)}{x-a}\right] = cf'(a)$$

$$y'(x) = cf'(x)$$

$$F(x) = f(x) + g(x)$$

$$F'(a) = \lim_{x\to a} \frac{[f(x)+g(x)] - [f(a)+g(a)]}{x-a} = \lim_{x\to a}\left[\frac{f(x)-f(a)}{x-a} + \frac{g(x)-g(a)}{x-a}\right]$$

$$= \lim_{x\to a}\left[\frac{f(x)-f(a)}{x-a}\right] + \lim_{x\to a}\left(\frac{g(x)-g(a)}{x-a}\right) = f'(a) + g'(a)$$

$$F'(x) = f'(x) + g'(x)$$

Ex. $y = \frac{4}{3}\pi r^3 \longmapsto y' = \frac{4}{3}\pi\left(\frac{d}{dr}r^3\right) = \frac{4}{3}\pi(3r^2) = 4\pi r^2$

Ex. $\frac{d}{dx}\left[x^8 + 12x^5 + 5\right] = \frac{d}{dx}(x^8) + 12\frac{d}{dx}(x^5) + \frac{d}{dx}(5) = 8x^7 + 12(5x^4) + 0 = 8x^7 + 60x^4$

Derivative Techniques

Math 2413
Dr. Kennedy
11 Feb. 2019
pg 16

$f(x) = x^2+1$ $\quad D(f(x)) = 2x$ $\quad f(x)g(x) = (x^2+1)x^{1/2} = x^{5/2} + x^{1/2}$

$g(x) = \sqrt{x} = x^{1/2}$ $\quad D(g(x)) = \frac{1}{2}x^{-1/2}$

$$D[f(x)g(x)] = D[f(x)]g(x) + f(x)D[g(x)]$$

$$= (2x)(x^{1/2}) + (x^2+1)(\tfrac{1}{2}x^{-1/2}) = 2x^{3/2} + x^2(\tfrac{1}{2}x^{-1/2}) + \tfrac{1}{2}x^{-1/2}$$

$$= 2x^{3/2} + \tfrac{1}{2}x^{3/2} + \tfrac{1}{2}x^{-1/2} = \tfrac{5}{2}x^{3/2} + \tfrac{1}{2}x^{-1/2}$$

$$D[f(x)g(x)] = D[x^{5/2} + x^{1/2}] = D[x^{5/2}] + D[x^{1/2}] = \tfrac{5}{2}x^{3/2} + \tfrac{1}{2}x^{-1/2}$$

$H(x) = \dfrac{3x+1}{x^2+2}$, $\quad f(x) = 3x+1$, $f'(x) = 3$, $\quad g(x) = x^2+2$, $g'(x) = 2x$

$$D[H(x)] = D\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2} = \frac{(3)(x^2+2) - (3x+1)(2x)}{(x^2+2)^2}$$