Curve Sketching

Ex. $f(x) = \dfrac{x^2}{x^2+3}$ Find local max/min; intervals where f inc/dec; domain; asymptotes. Give rough sketch of graph

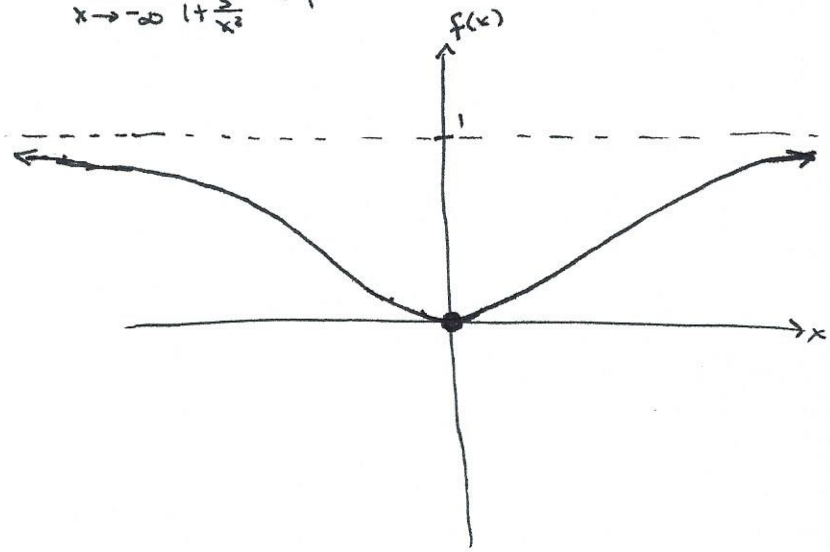
Domain: $\mathbb{R}$ = all reals

Asymptotes: no vertical

horiz. @ $y=1$

$$\lim_{x\to\infty} \frac{x^2}{x^2+3}\cdot\frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \lim_{x\to\infty}\frac{1}{1+\frac{3}{x^2}} = 1$$

$$\lim_{x\to-\infty}\frac{1}{1+\frac{3}{x^2}} = 1$$

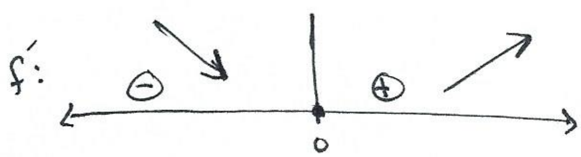


x-intercept: 0

y-intercept: 0

$$f'(x) = \frac{2x(x^2+3) - x^2(2x)}{(x^2+3)^2} = \frac{6x}{(x^2+3)^2}$$

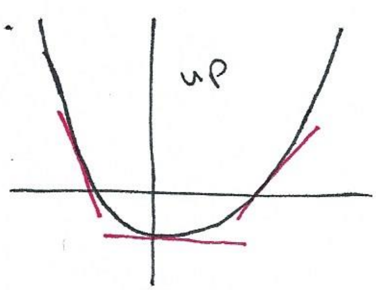
$\left.\begin{array}{l} f' \text{ exists on } \mathbb{R} \\ f'(x)=0 \Rightarrow 6x=0 \\ \Rightarrow x=0 \end{array}\right\}$ $x=0$ is the only crit #

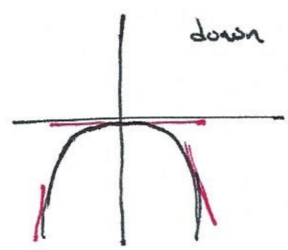


f dec on $(-\infty,0)$

f inc on $(0,\infty)$

Def. If the graph of f lies <u>above</u> all of its tangent lines on an interval I, then f is <u>concave up</u> on I.





Def. If the graph of f lies <u>below</u> all of its tangent lines on an interval I, then f is <u>concave down</u> on I.

Note: 1^{st} derivative f' gives information about increasing, decreasing, local max, local min

2^{nd} derivative f'' gives information about concavity and inflection points

Curve Sketching

Math 2413
Dr. Kennedy
28 Feb 2019
pg 38

Def. A point p on a curve $f(x)$ is an inflection point if f is continuous there and the curve changes concavity at p.

Concavity test: a) $f'' > 0$ on I, then f concave up on I

b) $f'' < 0$ on I, then f concave down on I

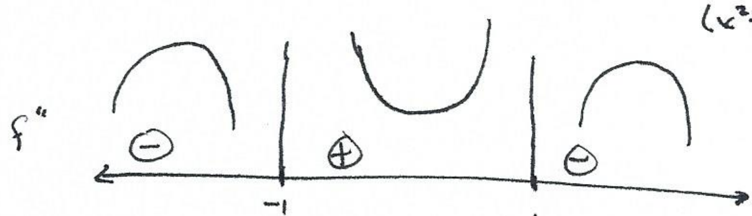
Ex. (cont) $f'(x) = \dfrac{6x}{(x^2+3)^2} \Rightarrow f''(x) = \dfrac{6(x^2+3)^2 - 6x(2)(x^2+3)(2x)}{(x^2+3)^4} = \dfrac{6(x^2+3) - 24x^2}{(x^2+3)^3} = \dfrac{6[x^2+3-4x^2]}{(x^2+3)^2}$

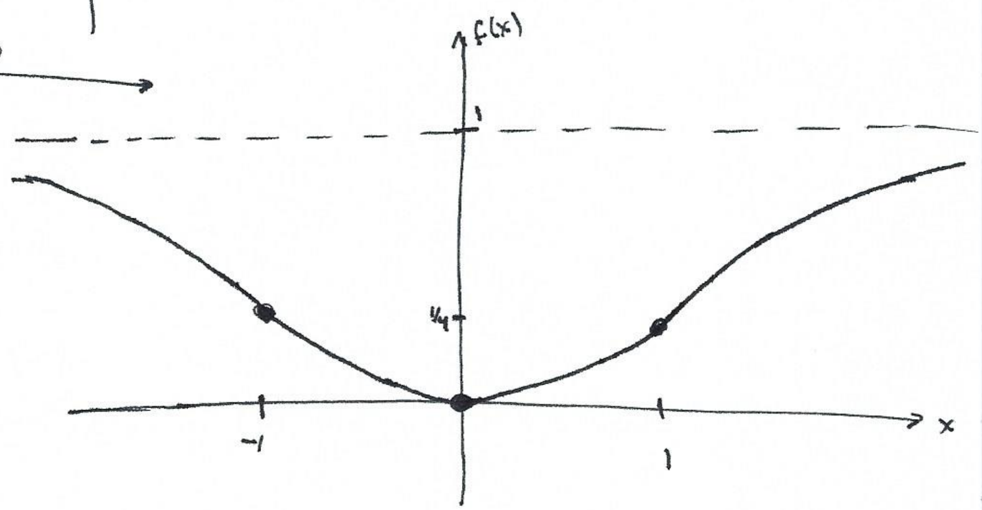
$= \dfrac{18(1-x^2)}{(x^2+3)^3}$

f'' defined everywhere

$f''(x) = 0 \Rightarrow 1 - x^2 = 0$

$\Rightarrow x = \pm 1$



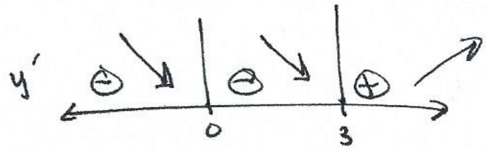


Ex. Discuss $y = x^4 - 4x^3$ wrt: concavity, inflection points, local max/min, intervals where f inc/dec. Sketch the graph of y.

- domain: $\mathbb{R}$
- no asymptotes

$y' = 4x^3 - 12x^2 = 4x^2(x-3)$

- y' defined everywhere
- $y'(x) = 0 \Rightarrow x = 0, 3$



$\Rightarrow y$ dec on $(-\infty, 3)$

y inc on $(3, \infty)$

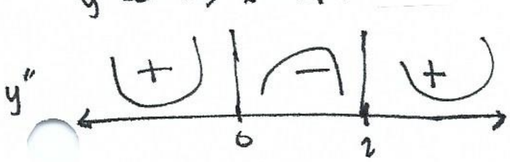
* local min:

$y(3) = 3^4 - 4 \cdot 3^3$

$= 27(3-4) = -27$

$y'' = 12x^2 - 24x = 12x(x-2)$

- y'' defined everywhere
- $y'' = 0 \Rightarrow x = 0, 2$



$\Rightarrow$ inflection points:

$(2, y(2)) = (2, -16)$

$(0, y(0)) = (0, 0)$

$\Rightarrow y$ concave up on $(-\infty, 0) \cup (2, \infty)$

y concave down on $(0, 2)$