

February 28, 2019

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* Pass out Exam and discuss *

Example

4) Find $\frac{d^2y}{dx^2}$ of $x^2 + y^2 = 25$.

$$x^2 + y^2 = 25$$

First find the first derivative

$$2x + 2y \frac{dy}{dx} = 0$$

$$2y \frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

Derive the first derivative to get the second

$$\frac{d^2y}{dx^2} = \frac{-(x)'y - x(y)'}{y^2} \quad \text{use quotient rule}$$

$$\frac{d^2y}{dx^2} = \frac{-y - x\left(\frac{dy}{dx}\right)}{y^2} \quad \rightarrow \text{we solved for } \frac{dy}{dx} = -\frac{x}{y}.$$

Sub it in

$$\frac{d^2y}{dx^2} = \frac{-y - x\left(-\frac{x}{y}\right)}{y^2}$$

$$\frac{d^2y}{dx^2} = \frac{-y + \frac{x^2}{y}}{y^2} = -\frac{\frac{y^2}{y} + \frac{x^2}{y}}{y^2} = -\frac{y^2 + x^2}{y^3} \quad \text{substitute } x^2 + y^2 = 25$$

$$\boxed{\frac{d^2y}{dx^2} = -\frac{25}{y^3}}$$