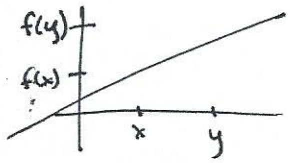
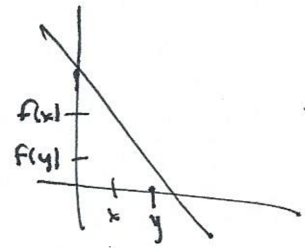
Curve Sketching

Def. The function f is <u>increasing</u> on an interval if for <u>$x<y$</u> with x, y in the interval, <u>$f(x)<f(y)$</u>.

Def. The function f is <u>decreasing</u> on an interval if for <u>$x<y$</u> with x, y in the interval, <u>$f(x)>f(y)$</u>



increasing

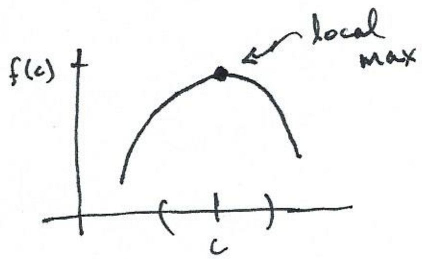


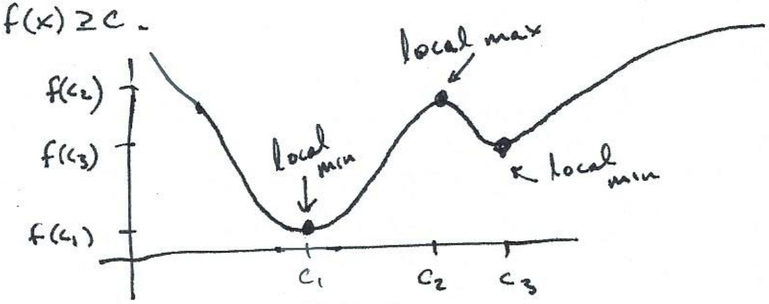
decreasing

Let f continuous on its domain.

Def. f has a <u>local (relative) max</u> at c if there is an open interval containing c such that for each x in the interval, $f(x) \le f(c)$.

Def. f has a <u>local (relative) min</u> at c if there is an open interval containing c such that for each x in the interval, $f(x) \ge c$.





f decreasing on $(-\infty, c_1) \cup (c_2, c_3)$

means union

f increasing on $(c_1, c_2) \cup (c_3, \infty)$

If f is differentiable on some interval

a) if $f' > 0$ on that interval, then f is increasing on that interval

b) if $f' < 0$ on that interval, then f is decreasing on that interval.

Curve Sketching

Ex. $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$

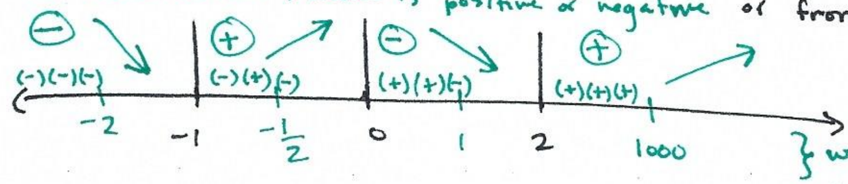
- Domain : $\mathbb{R}$ = all reals
- no horizontal or vertical asymptotes.

Find derivative and critical numbers

$$f'(x) = 12x^3 - 12x^2 - 24x = 12x\left[x^2 - x - 2\right] = 12x(x+1)(x-2)$$

$$f'(x) = 0 = 12x(x+1)(x-2) \longrightarrow x = 0, -1, 2$$ } these are the only places where f can change from increasing to decreasing or from dec. to inc.

Pick test numbers in the intervals below, and check whether f value is positive or negative



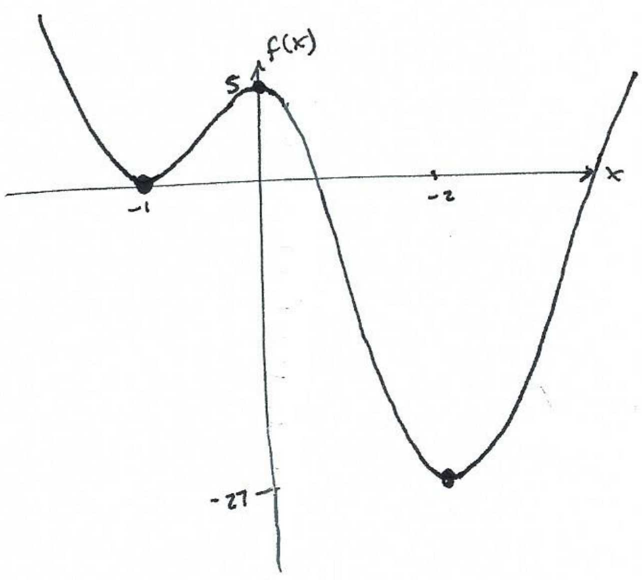
} we can choose any number in these intervals

f is decreasing on $(-\infty, -1) \cup (0, 2)$

increasing on $(-1, 0) \cup (2, \infty)$

<u>local minima of f</u> at $x = -1$ and $x = 2$: $f(-1) = 0$, $f(2) = -27$

<u>local maximum of f</u> at $x = 0$: $f(0) = 5$



Curve Sketching

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Dr. Kennedy
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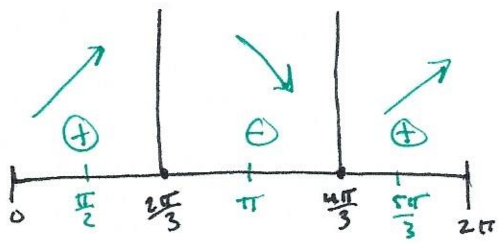
Ex. Find local max and local min of $g(x) = x + 2\sin(x)$, $0 \le x \le 2\pi$

→ find critical numbers:

$g'(x) = 1 + 2\cos(x) = 0 \implies \cos(x) = -\frac{1}{2} \implies x = \frac{2\pi}{3}, \frac{4\pi}{3}$



g increasing on $\left[0, \frac{2\pi}{3}\right) \cup \left(\frac{4\pi}{3}, 2\pi\right]$

decreasing on $\left(\frac{2\pi}{3}, \frac{4\pi}{3}\right)$

g has local max at $x = \frac{2\pi}{3}$: $g\left(\frac{2\pi}{3}\right) = \frac{2\pi}{3} + \sqrt{3} \approx 3.83$

g has local min at $x = \frac{4\pi}{3}$: $g\left(\frac{4\pi}{3}\right) = \frac{4\pi}{3} - \sqrt{3} \approx 2.46$