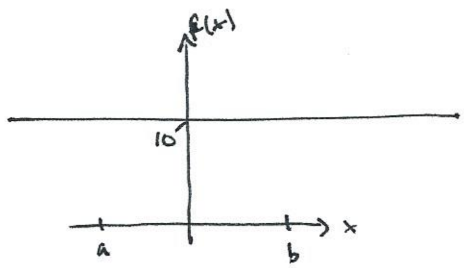
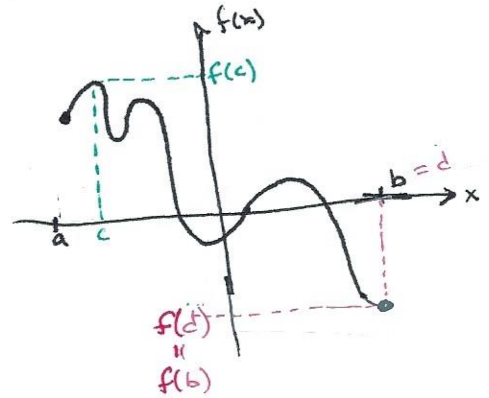
Extreme Value Theorem

Thm. Suppose f is continuous on $[a,b]$. Then f attains a maximum value $f(c)$ and a minimum value $f(d)$ at some numbers c, d in $[a,b]$.

f has a <u>maximum</u> or <u>absolute maximum</u> $f(c)$ on the set D if $f(x) \le f(c)$ for each x in D.

f has a <u>minimum</u> or <u>absolute minimum</u> $f(d)$ on the set D if $f(x) \ge f(d)$ for each x in D.



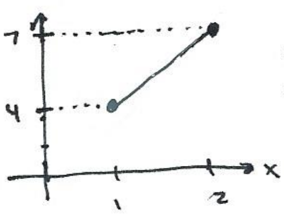


if $x \in [a,b]$, then $f(x) = 10 \le 10$

and $f(x) = 10 \ge 10$

So 10 is the max and min

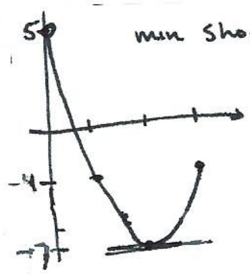
Ex. $f(x) = 3x + 1$ on $[1,2]$



min: $f(1) = 4$

max: $f(2) = 7$

Ex. $f(x) = 3x^2 - 12x + 5$ on $[0,3]$



min should happen when $f'(x) = 0$,

or $6x - 12 = 0 \mapsto x = 2$

min: $f(2) = -7$

max: $f(0) = 5$

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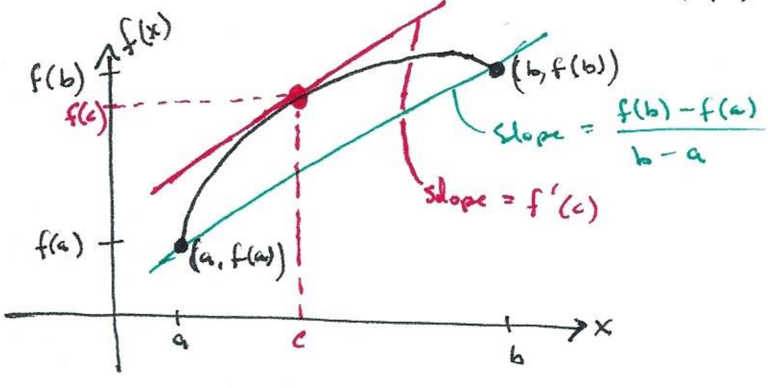
Mean Value Theorem

Thm. Suppose the function f satisfies 1) f continuous on $[a,b]$

2) f differentiable on (a,b).

Then there is a number c in (a,b) such that $f'(c) = \dfrac{f(b)-f(a)}{b-a}$



Ex. Let $f(x) = x^3 - x$. Find a number c in $(a,b) = (0,2)$ such that $f'(c) = \dfrac{f(b)-f(a)}{b-a}$

$f(a) = f(0) = 0$

$f(b) = f(2) = 6$

$b-a = 2$

so $\dfrac{f(b)-f(a)}{b-a} = \dfrac{6-0}{2} = 3$

1) f continuous on $[a,b]$ ✓

2) f differentiable on (a,b) ✓

$$f'(c) = 3c^2 - 1 = \frac{f(b)-f(a)}{b-a} = 3$$

$$3c^2 - 1 = 3$$

$$3c^2 = 4$$

$$c^2 = \frac{4}{3}$$

$c = \pm \dfrac{2}{\sqrt{3}}$, but $\dfrac{-2}{\sqrt{3}}$ not in, so

$$c = \frac{2}{\sqrt{3}}$$