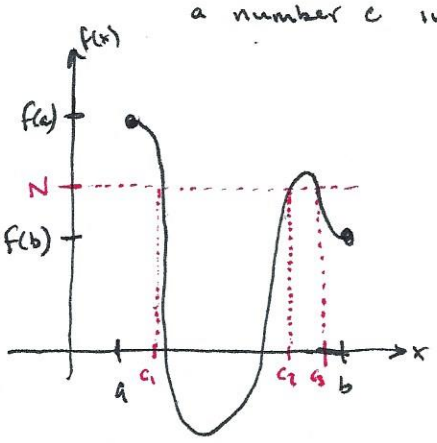
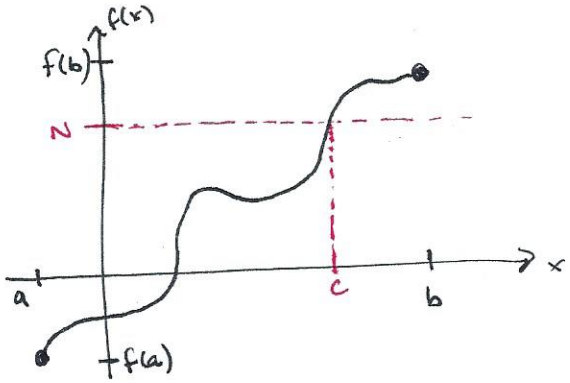
Intermediate Value Theorem (IVT)

Thm. Suppose f is continuous on the closed interval $[a,b]$.

If N is any number between $f(a)$ and $f(b)$ $(f(a) \neq f(b))$, then there exists a number c in (a,b) such that $f(c) = N$.





Ex. Show that there is a root of $f(x) = 4x^3 - 6x^2 + 3x - 2 = 0$ between 1 and 2.

f is continuous on $\mathbb{R}$ since f is a polynomial, so f is continuous on $[1,2]$.

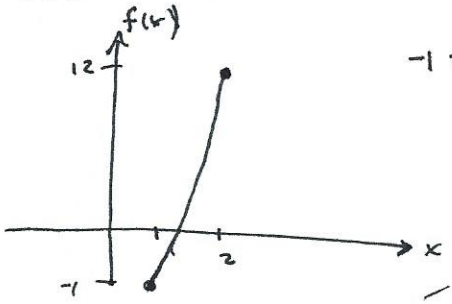
$f(1) = 4 - 6 + 3 - 2 = -1$

$f(2) = 32 - 24 + 6 - 2 = 12$

$\longmapsto f(1) \neq f(2)$

$-1 = f(1) < 0 < f(2) = 12$

By IVT, there exists a number $c \in (1,2)$ such that $f(c) = 0$



Ex. Prove that $x = \cos(x)$ has at least one solution

Let $f = x - \cos(x)$. Then $f(0) = -1$ and $f(\pi/2) = \pi/2$.

Since f is continuous on $[0, \pi/2]$ and $f(0) < 0 < f(\pi/2)$, there is a number $c \in (0, \pi/2)$ such that $f(c) = 0$.