Related Rates

Math 2413
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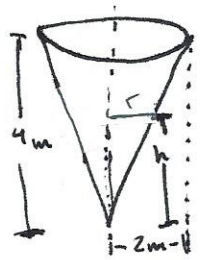
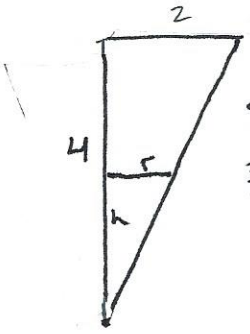
Ex. 

A water tank has the shape of an inverted circular cone with base radius 2 m and height 4 m. If water is being pumped into the tank at a rate of 2 $\frac{m^3}{min}$, find the rate at which the water level is rising when the water is 3 m deep.

Volume of a cone: $V = \frac{\pi}{3} r^2 h$, $V = V(t)$

Given: $\frac{dV}{dt} = 2 \frac{m^3}{min}$

want to know: $\frac{dh}{dt}$ @ $h = 3m$



Similar triangles

$\frac{2}{4} = \frac{r}{h} \mapsto r = \frac{h}{2} \longmapsto V = \frac{\pi}{3}\left(\frac{h}{2}\right)^2 h = \frac{\pi}{3}\frac{h^2}{4} h = \frac{\pi}{12} h^3$

$\rightarrow V = \frac{\pi}{12} h^3$

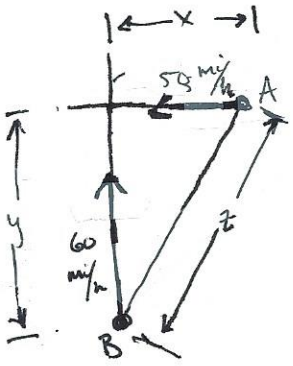
$2 = \frac{dV}{dt} = \frac{\pi}{12}\frac{d}{dt}(h^3) = \frac{\pi}{12}\left(3h^2 \frac{dh}{dt}\right) = \frac{\pi}{4} h^2 \frac{dh}{dt}$

$\rightarrow 2 = \frac{\pi}{4} h^2 \frac{dh}{dt}$

$\frac{8}{\pi h^2} = \frac{dh}{dt}$, $h \neq 0$

$\Rightarrow$ when $h = 3m$, $\frac{dh}{dt} = \frac{8}{\pi(3)^2} \frac{m}{s} = \frac{8}{9\pi} \frac{m}{s}$

Ex. Car A is travelling west at 50 $\frac{mi}{h}$ and car B is travelling north at 60 $\frac{mi}{h}$. Both are headed for the intersection of the two roads. At what rate are the cars approaching each other when car A is .3 mi and car B is .4 mi from the intersection.



$x^2 + y^2 = z^2 \longmapsto 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$x\frac{dx}{dt} + y\frac{dy}{dt} = z\frac{dz}{dt}$

$x = x(t)$, $y = y(t)$, $z = z(t)$ } given / want to know

at the moment in time when $x = .3$, $y = .4$

$z^2 = (.3)^2 + (.4)^2 = .09 + .16 = .25$

$\rightarrow z = 0.5$

$(.3)(-50) + (.4)(-60) = .5\frac{dz}{dt}$

$-15 - 24 = -39 = \frac{1}{2}\frac{dz}{dt} \longmapsto \frac{dz}{dt} = -78$

$\Rightarrow$ when car A is .3 mi and car B is .4 mi from the intersection, $\frac{dz}{dt} = -78 \frac{mi}{hr}$