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3.4. The Chain Rule

Theorem 3.4.1 The Chain Rule

If $y = f(u)$ is a differentiable function of u and $u = g(x)$ is a differentiable function of x, then $y = f(g(x))$ is a differentiable function of x and

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

or equivalently,

$$\frac{d}{dx}(f(g(x))) = f'(g(x))g'(x)$$

Example. If $f(x) = (2x+1)^{60}$.

$f'(x) = f'(g(x))g'(x)$ · chain rule.

let $2x+1 = g(x)$. Then $g'(x) = 2$.

So $f'(x) = 2(2x+1)^{60-1}$

$f'(x) = 2(2x+1)^{59}$.

Theorem 3.4.2

Let f be a differentiable function of x.

1) The General Power Rule: If $y = (f(x))^n$, where n is a rational number, then

$$(f(x)^n)' = n\, f(x)^{n-1} f(x)'$$

2) Transcendental Functions and the Chain Rule:

$(\sin f(x))' = \cos(f(x))f'(x)$, $(\cos f(x))' = -\sin(f(x))f'(x)$,

$(\tan f(x))' = \sec^2(f(x))f'(x)$, $(\cot f(x))' = -\csc^2(f(x))f'(x)$,

$(\sec f(x))' = \sec(f(x))\tan(f(x))f'(x)$,

$(\csc f(x))' = -\csc(f(x))\cot(f(x))f'(x)$, and

$(e^{f(x)})' = e^{f(x)} f'(x)$