Theorem 3.4.1

The Chain Rule

Let \( y = f(u) \) be a differentiable function of \( u \) and let \( u = g(x) \) be a differentiable function of \( x \). Then \( y = f(g(x)) \) is a differentiable function of \( x \) and

\[
\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}
\]

or equivalently,

\[
\frac{d}{dx} (f(g(x))) = f'(g(x))g'(x)
\]

Example. If \( f(x) = (2x+1)^4 \),

\[f'(x) = 4(2x+1)^3 \cdot (2x+1)' \cdot (2x+1)'
\]

Let \( 2x+1 = g(x) \). Then \( g'(x) = 2 \).

So \( f'(x) = 2(2x+1)^3 \cdot 2 \cdot g'(x) \)

\[f'(x) = 2(2x+1)^3 \cdot 2 \cdot 2 \]

Theorem 3.4.2

Let \( f \) be a differentiable function of \( x \).

1) The General Power Rule: If \( y = (f(x))^n \), where \( n \) is a rational number, then

\[ (f(x))^n\)' = n f(x)^{n-1} f'(x) \]

2) Transcendental Functions and the Chain Rule:

- \( (\sin f(x))' = \cos f(x)f'(x) \), \( (\cos f(x))' = -\sin f(x)f'(x) \),
- \( (\tan f(x))' = \sec^2 f(x)f'(x) \), \( (\cot f(x))' = -\csc^2 f(x)f'(x) \),
- \( (\sec f(x))' = \sec f(x)\tan f(x)f'(x) \), \( (\csc f(x))' = -\csc f(x)\cot f(x)f'(x) \),
- \( (e^{f(x)})' = e^{f(x)}f'(x) \), and