Related Rates

Math 2413
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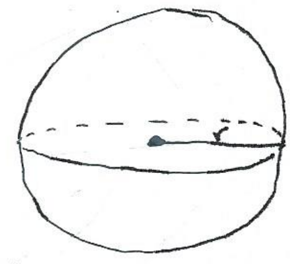
Ex. 1 Air is being pumped into a spherical balloon so that its volume increases at a rate of $100 \frac{cm^3}{s}$. How fast is the radius of the balloon increasing when its diameter is 50 cm?

Volume of sphere $= \frac{4}{3}\pi r^3 = V$



$V = V(t)$ cm^3/s

$r = r(t)$ cm

Given: $\frac{dV}{dt} = 100$ want to know: $\frac{dr}{dt}$ when $r = \frac{50}{2}$ cm $= 25$ cm

* $r = 25$ at only one moment in time.

$$\frac{dV}{dt} = \frac{4}{3}\pi \frac{d}{dt}\left[r^3\right] = \frac{4}{3}\pi\left(3r^2\frac{dr}{dt}\right) = 4\pi r^2 \frac{dr}{dt}$$

when $r = 25$: $\frac{dV}{dt} = 100 = 4\pi(25)^2\frac{dr}{dt}$

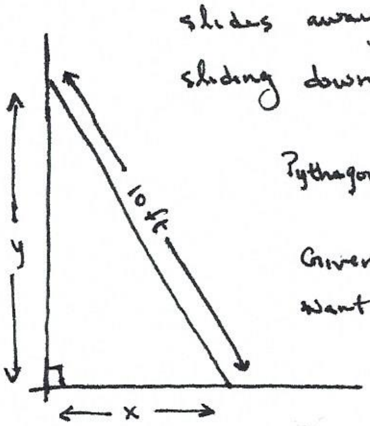
$$25 = \pi \cdot 25^2 \frac{dr}{dt}$$

$$1 = 25\pi \frac{dr}{dt}$$

$$\frac{1}{25\pi} = \frac{dr}{dt}$$

$$\boxed{\frac{dr}{dt} = \frac{1}{25\pi}\ \frac{cm}{s} \text{ when } r = 25}$$

Ex. 2 A ladder 10 ft tall rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 ft/s, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 6 ft from the wall?



Pythagorean Theorem: $x^2 + y^2 = 10^2$, $x = x(t)$, $y = y(t)$, $\frac{dx}{dt} > 0$ (x increasing), $\frac{dy}{dt} < 0$ (y decreasing)

Given: $\frac{dx}{dt} = 1$

want to know: $\frac{dy}{dt}$ when $x = 6$.

* when $x = 6$, $6^2 + y^2 = 100 \mapsto y^2 = 100 - 36 = 64 \longrightarrow y = 8$ (since -8 does not make physical sense here)

$$x^2 + y^2 = 100$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$x\frac{dx}{dt} + y\frac{dy}{dt} = 0$$

when $x = 6$: $(6)(1) + (8)\frac{dy}{dt} = 0$

$$8 \cdot \frac{dy}{dt} = -6$$

$$\frac{dy}{dt} = \frac{-6}{8} = \frac{-3}{4}$$

$$\boxed{\frac{dy}{dt} = \frac{-3}{4}\ \frac{ft}{s} \text{ when } x = 6}$$