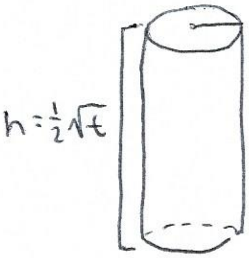
February 14, 2014 HAPPY VALENTINE'S

4. The radius of a right circular cylinder is given by $\sqrt{t+2}$ and its height is $\frac{1}{2}\sqrt{t}$, where t is time in seconds and the dimensions are in inches. Find the rate of change of the volume with respect to time.



volume $= \pi r^2 h$

$= \pi(\sqrt{t+2})^2(\frac{1}{2}\sqrt{t})$ sub for $r=\sqrt{t+2}$ and $h=\frac{1}{2}\sqrt{t}$

$= \pi(t+2)(\frac{1}{2}\sqrt{t})$

$= \frac{\pi}{2}(t+2)(\sqrt{t})$ simplify

$= \frac{\pi}{2}(t\sqrt{t}+2\sqrt{t})$

$= \frac{\pi}{2}(t(t^{\frac{1}{2}})+2(t^{\frac{1}{2}}) \rightarrow V(t)=\frac{\pi}{2}(t^{\frac{3}{2}}+2t^{\frac{1}{2}})$

Rate of change implies derivative!

for $V(t)=\frac{\pi}{2}(t^{\frac{3}{2}}+2t^{\frac{1}{2}})$

$V'(t)=\frac{\pi}{2}[t^{\frac{3}{2}}+2t^{\frac{1}{2}}]'$

$=\frac{\pi}{2}((\frac{3}{2})t^{\frac{3}{2}-1}+2(\frac{1}{2})t^{\frac{1}{2}-1})$

$V'(t)=\frac{\pi}{2}(\frac{3}{2}t^{\frac{1}{2}}+t^{-\frac{1}{2}})$

Definition 3.3.3

1. You can obtain an acceleration function by differentiating a velocity function. The function $a(t)$ is the second derivative of $s(t)$ and is denoted by $s''(t)$.

2. The second derivative is an example of a higher-order derivative. In general, the derivative is denoted by

$$y^{(n)} \quad f^{(n)}(x) \quad \frac{d^n y}{dx^n} \quad D_x^n[y]$$

Example 3.3.4

1. Given the position function $s(t) = 0.81t^2 + 2$, where t is in seconds and $s(t)$ is in meters, what if the acceleration after $t = 5$ seconds?

acceleration is the second derivative of position so, find $s''(t)$.

$$s(t) = 0.81t^2 + 2$$

$$s'(t) = 0.81(2)t^{2-1}$$

$$s'(t) = 1.62t^1$$

derive again to find $s''(t) = a(t)$

$$s''(t) = (1.62t)'$$

$$s''(t) = 1.62.$$

Acceleration after $t = 5$,

$$\boxed{a(5) = 1.62 \, m/s^2}$$

→ since $a(t) = 1.62$ for any t.

2. Find $f^3(x)$ is $f''(x) = 2 - \frac{2}{x}$.

$f^3(x)$ is the derivative of $f''(x)$.

So $f^3(x) = \left(2 - \frac{2}{x}\right)'$

$$= (2)' - \left(\frac{2}{x}\right)'$$

$$= 0 - (2(x)^{-1})'$$

$$= 0 - (2)(-1)(x)^{-1-1}$$

$$\boxed{f^3(x) = \frac{2}{x^2}}$$