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Find the derivative

b) $l(x) = xe^x(5+4x)$

$l'(x) = (xe^x(5+4x))'$ - use the product rule

If $l(x) = f(x)g(x)$, $l'(x) = f'(x)g(x) + f(x)g'(x)$.

let $\boxed{f(x) = xe^x}$ and $\boxed{g(x) = (5+4x)}$

so $f'(x) = (x)'(e^x) + (x)(e^x)'$ and $g'(x) = (5+4x)'$

$\boxed{f'(x) = e^x + xe^x}$ and $\boxed{g'(x) = 4}$

Substitute the values into $l'(x)$

$l'(x) = f'(x)g(x) + f(x)g'(x)$

$$\boxed{l'(x) = (e^x + xe^x)(5+4x) + (xe^x)(4)}$$

c) $h(x) = x\left(c - \frac{4}{x+3}\right)$, where c is a constant

$h'(x) = \left[x\left(c - \frac{4}{x+3}\right)\right]'$ → use product rule for $f(x) = x$ and $g(x) = c - \frac{4}{x+3}$

$h'(x) = (x)'\left(c - \frac{4}{x+3}\right) + (x)\underbrace{\left(c - \frac{4}{x+3}\right)'}$ $c' = 0$, but we need to use quotient rule for $\frac{4}{x+3} = \frac{p(x)}{g(x)}$, $\left(\frac{4}{x+3}\right)' = \frac{p'(x)g(x) - p(x)g'(x)}{(g(x))^2}$

$h'(x) = 1\left(c - \frac{4}{x+3}\right) + x\left(0 - \frac{(4)'(x+3) - 4((x+3)')}{(x+3)^2}\right)$

$h'(x) = \left(c - \frac{4}{x+3}\right) + x\left(-\frac{0(x+3) - 4(1)}{(x+3)^2}\right)$

$h'(x) = c - \frac{4}{x+3} + x\left(-\frac{-4}{(x+3)^2}\right)$

$$\boxed{h'(x) = c - \frac{4}{x+3} + \frac{4x}{(x+3)^2}}$$

Quotient Rule

if $h(x) = \frac{f(x)}{g(x)}$, then

$h'(x) = \left(\frac{f(x)}{g(x)}\right)'$ where

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

2. Find $f'(0)$ if $f(x) = \frac{\cos x}{e^x}$

$f'(x) = \left(\frac{\cos x}{e^x}\right)'$ → use the quotient rule

$= \frac{(\cos x)'(e^x) - (\cos x)(e^x)'}{(e^x)^2}$

Recall
$\sin 0 = 0$
$e^0 = 1$
$\cos 0 = 1$

Also,
$(\cos x)' = -\sin x$
$(e^x)' = e^x$

$f'(x) = \frac{(-\sin x)(e^x) - (\cos x)(e^x)}{e^{2x}}$

substitute for $x = 0$

$f'(0) = \frac{(-\sin 0)(e^0) - (\cos 0)(e^0)}{e^{20}} = \frac{(-\cancel{0}^{0})(1) - (1)(1)}{1}$

$f'(0) = \frac{-1}{1}$, $\boxed{f'(0) = -1}$

3. Find the equation of the tangent line(s) to the graph of $g(x) = \frac{x+1}{x-1}$ that are parallel to the line $2y + x = 6$.

parallel implies $g(x)$ and y have the same slope.

∘ derive $g'(x)$ and y'.

$g'(x) = \left(\frac{x+1}{x-1}\right)'$

$= \frac{(x+1)'(x-1) - (x+1)(x-1)'}{(x-1)^2}$

$= \frac{(1)(x-1) - (x+1)(1)}{(x-1)^2} = \frac{x-1-(x+1)}{(x-1)^2}$

$= \frac{x-1-x-1}{(x-1)^2}$ → $\boxed{g'(x) = \frac{-2}{(x-1)^2}}$

$2y + x = 6$

$y = 3 - \frac{x}{2}$

$y' = \left(3 - \frac{x}{2}\right)' = \left(3 - \frac{1}{2}x\right)'$

$= (3)' - \frac{1}{2}(x)'$, $\boxed{y' = -\frac{1}{2}}$

These must be equal

we found $g'(x) = -\frac{2}{(x-1)^2}$ and $y' = -\frac{1}{2}$.

They should be the same since the lines are parallel.

The negatives cancel.

So $(+)\frac{2}{(x-1)^2} = (+)\frac{1}{2}$. Cross multiply

$2(2) = 1(x-1)^2$ → Solve for x.

$4 = (x-1)^2$

$\sqrt{4} = \sqrt{(x-1)^2}$

$\pm 2 = x - 1$. So $x = 1 \pm 2$.

$x = 1 + 2 = 3$

and $1 - 2 = -1$.

$\boxed{x = -1, 3}$

This is where the lines will meet

We have 2 x-values, so there are 2 tangent lines.

For $x = 3$, $g(3) = \frac{3+1}{3-1} = 2$. The point is $(3, 2)$

So $y - y_1 = m(x - x_1) \rightarrow y - 2 = -\frac{1}{2}(x - 3)$

$y - 2 = -\frac{1}{2}x + \frac{3}{2}$

$\boxed{y = -\frac{1}{2}x + \frac{7}{2}}$

For $x = -1$, $g(-1) = \frac{3+(-1)}{3-(-1)} = 0$, The point is $(-1, 0)$

So $y - y_1 = m(x - x_1) \rightarrow y - 0 = -\frac{1}{2}(x - (-1))$

$\boxed{y = -\frac{1}{2}x - \frac{1}{2}}$

Here are the equations of the tangent lines.