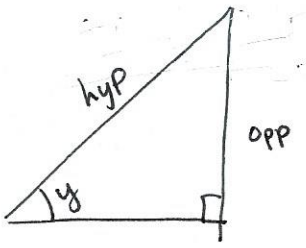
Inverse Trig Functions

Want $\frac{dy}{dx} = y'$ for $\arcsin(x) = y$

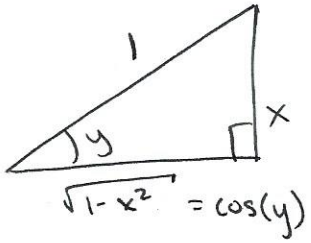
$\sin(\arcsin(x)) = \sin(y)$

$x = \sin(y)$

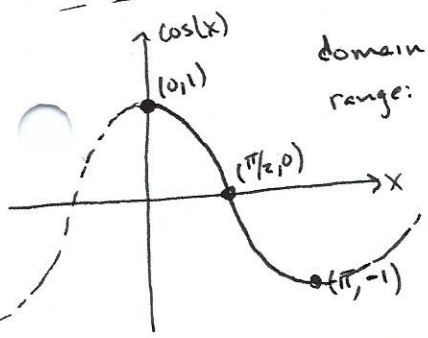
$1 = \cos(y)y' \longmapsto y' = \frac{1}{\cos(y)} = \sec(y)$



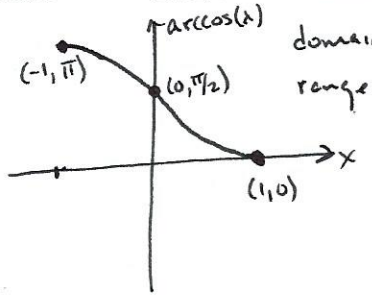
$x = \sin(y) = \frac{opp}{hyp} = \frac{x}{1}$



$$\Longrightarrow y' = \frac{dy}{dx} = \frac{d}{dx}\left[\arcsin(x)\right] = \frac{1}{\sqrt{1-x^2}}$$

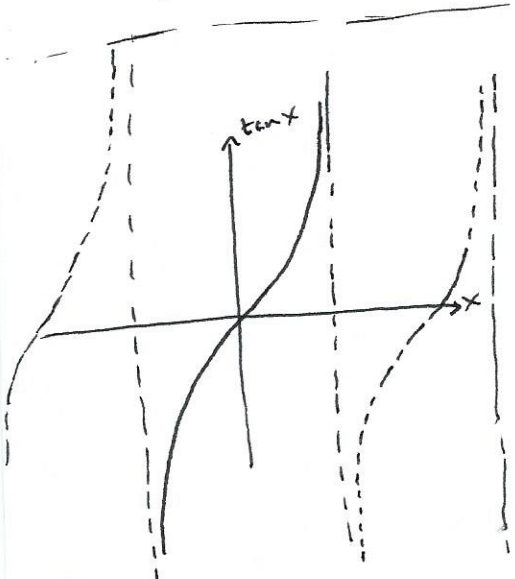


domain: $[0, \pi]$
range: $[-1, 1]$

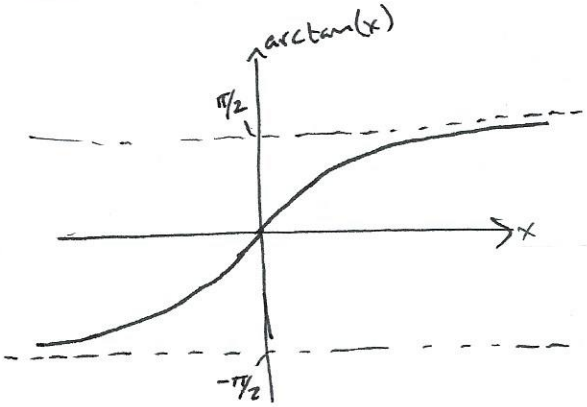


domain: $[-1, 1]$
range: $[0, \pi]$

$$D\left[\arccos(x)\right] = \frac{-1}{\sqrt{1-x^2}}$$



domain: $(-\pi/2, \pi/2)$
range: $(-\infty, \infty)$



domain: $(-\infty, \infty)$
range: $(-\pi/2, \pi/2)$

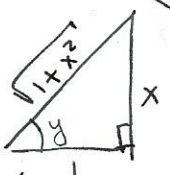
Want $\frac{dy}{dx}$ for $y = \arctan(x)$

$\tan(y) = x$

$\sec^2(y)y' = 1$

$y' = \frac{1}{\sec^2(y)} = \cos^2(y)$

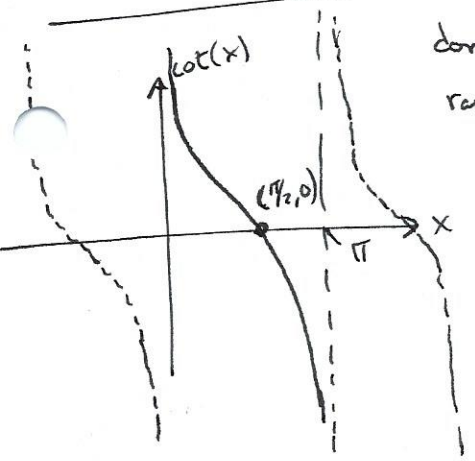
$x = \tan(y) = \frac{opp}{adj}$



$\cos(y) = \frac{1}{\sqrt{1+x^2}}$

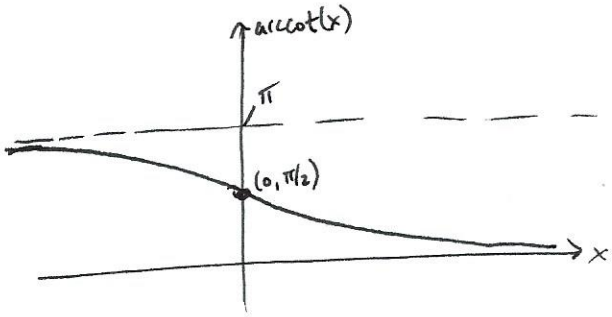
$\cos^2(y) = \frac{1}{1+x^2} = y' = \frac{d}{dx}\left[\arctan(x)\right]$

Inverse Trig Functions

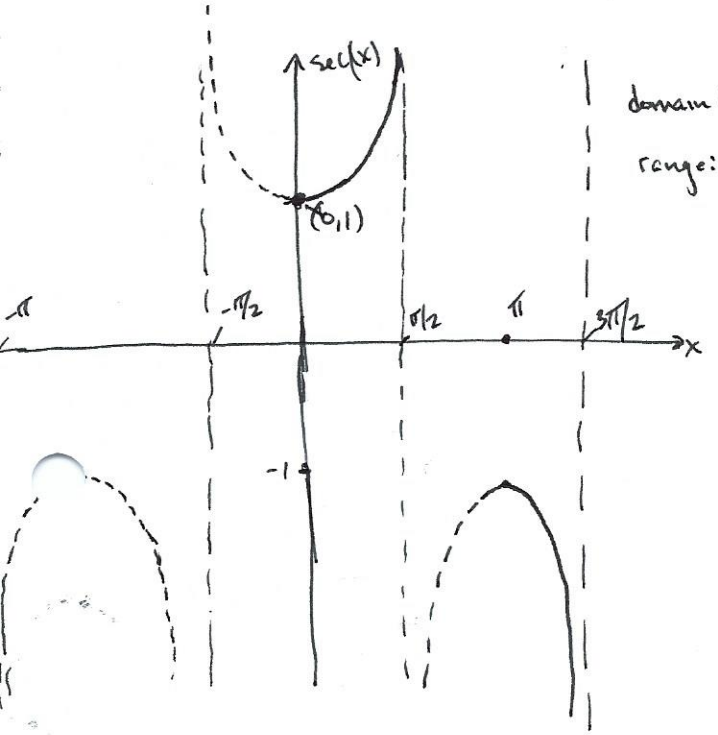


domain: $(0, \pi)$

range: $(-\infty, \infty)$

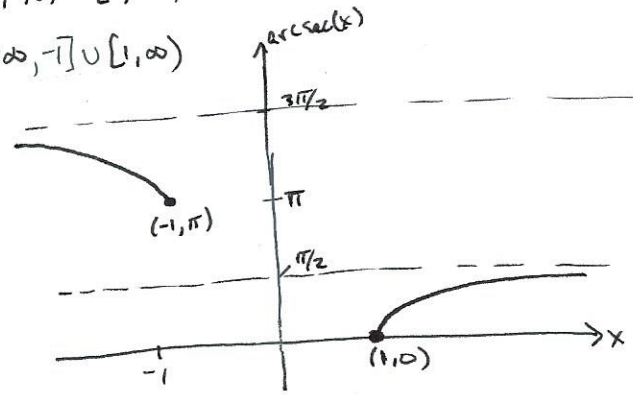


$$D[\operatorname{arccot}(x)] = \frac{-1}{1+x^2}$$



domain: $[0, \pi/2) \cup [\pi, 3\pi/2)$

range: $(-\infty, -1] \cup [1, \infty)$



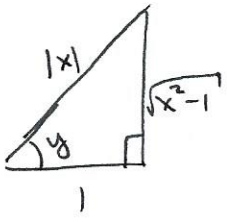
Want $\frac{dy}{dx}$ for $y = \operatorname{arcsec}(x)$

$$\sec(y) = \sec[\operatorname{arcsec}(x)] = x$$

$$\sec(y)\tan(y)\,y' = 1$$

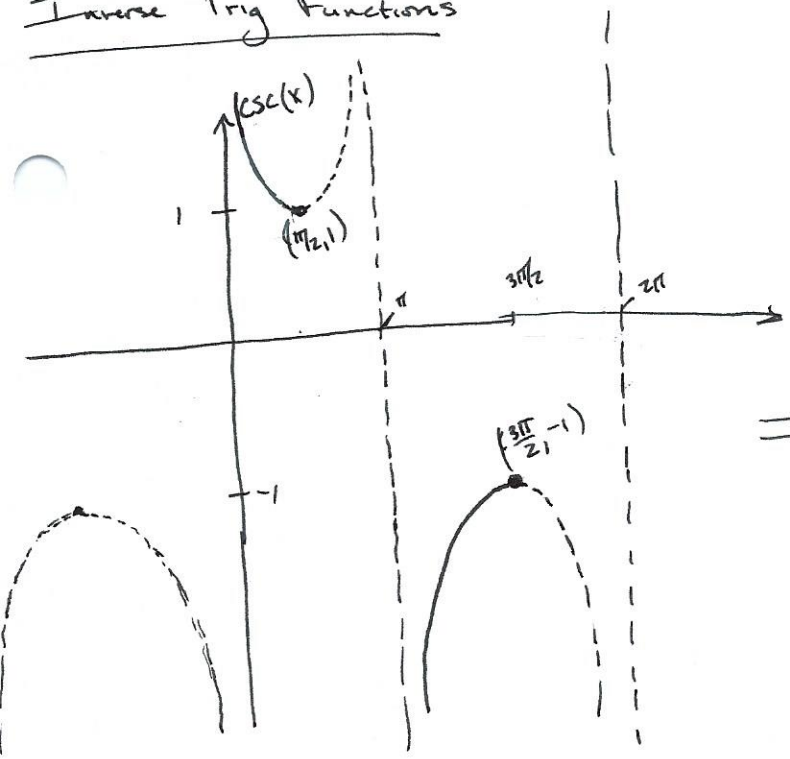
$$y' = \frac{1}{\sec(y)\tan(y)}$$

$$\sec(y) = \frac{1}{\cos(y)} = x$$



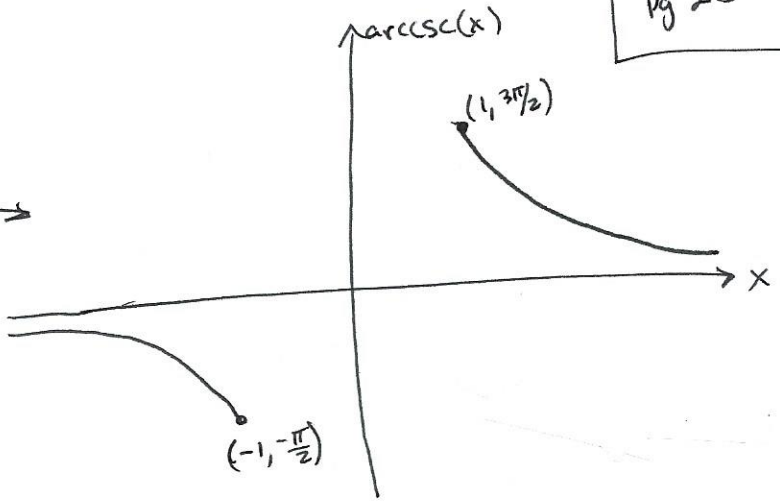
$$\Rightarrow D[\operatorname{arcsec}(x)] = \frac{1}{|x|\sqrt{x^2-1}}$$

Inverse Trig Functions



domain: $(0, \pi/2] \cup (\pi, 3\pi/2]$

range: $(-\infty, -1] \cup [1, \infty)$



domain: $(-\infty, -1] \cup [1, \infty)$

range:

Ex. domain $[-1, 1]$

$y = \sin(x) + \arcsin(x)$

$y' = \cos(x) + \frac{1}{\sqrt{1-x^2}}$

$y = [\sin(x)][\arcsin(x)]$

$y' = [D(\sin(x))]\arcsin(x) + \sin(x)[D(\arcsin(x))]$

$= \cos(x)\arcsin(x) + \sin(x) \cdot \frac{1}{\sqrt{1-x^2}}$

$y = \sin(\arcsin(x)) = x$

$y' = \cos(\arcsin(x))\, D(\arcsin(x))$

$= \cos(\arcsin(x)) \cdot \frac{1}{\sqrt{1-x^2}} \stackrel{?}{=} 1$