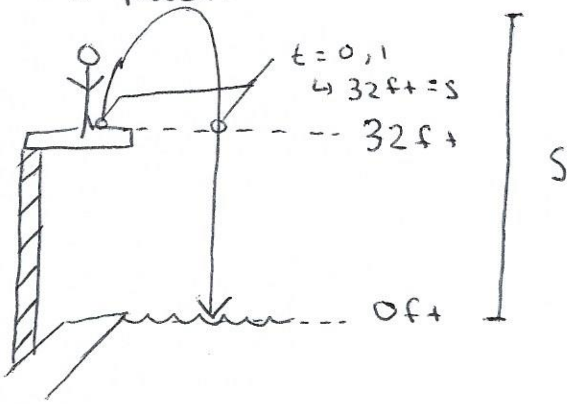
February 12, 2019

2. At time ~~t=0~~, a diver jumps from a platform diving board that is 32 feet above the water. Because the initial velocity of the diver is 16 feet per second, the position of the diver is $s(t) = -16t^2 + 16t + 32$, where s is measured in feet and t is measured in seconds. When does the diver hit the water? What is the diver's velocity at impact?



Hitting the water is at 0ft above the water.

So $S(t) = 0 = -16t^2 + 16t + 32$

$0 = +16(-t^2 + t + 2)$

$0 = -16(t^2 - t - 2)$

$0 = -16(t+1)(t-2)$

$0 \neq -16$ or $0 = t+1$ or $0 = t-2$.

Therefore $t = -1, 2$. Time cannot be negative! So $t = 2$.

$\boxed{\text{The diver hits the water after 2 seconds}}$

velocity: $s'(t) = -32t + 16 = v(t)$

at 2 seconds we have impact $(t=2)$

So $s'(t) = v(t) = -32(2) + 16$ when $t = 2$

$\boxed{v(2) = -48 \text{ feet/sec}}$

Be sure to use the correct units given from the problem

3. Use f(x) to answer the following questions.

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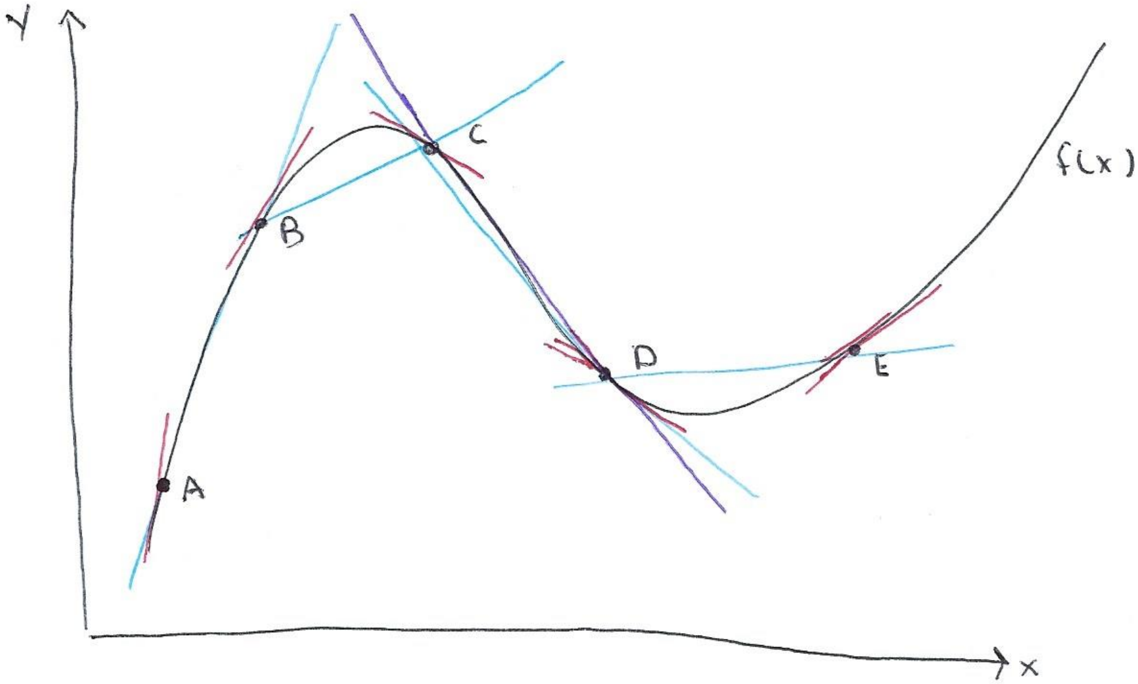

(a) Between which two consecutive points is the average rate of change of the function greatest?

Secant lines (slope)

Recall, average rate of change is $\frac{f(x_2)-f(x_1)}{x_2-x_1}$. (The slope)
CD and DE are both negative, so they are not the greatest rate of change. Our only choices are the slopes between A and B and B and C. AB is greater because it is steeper (moves up faster) than BC. The answer is $\boxed{\text{A and B}}$

(b) At which point is the instantaneous rate of change the greatest?

tangent lines (slope at the point)

C and D have negative rates, whereas A, B, E have positive rates. A has the steepest slope of tangent line, therefore, $\boxed{A}$ has the greatest instantaneous rate of change.

(c) Sketch a tangent line to f(x) between C and D such that the slope of the tangent line is the same as the average rate of change of f(x).

Same slope implies parallel. We just need to shift f(x) until it touches the line between C and D exactly once. (since tangents only share one point of the function)

3.3 Product and Quotient Rules and Higher-order derivatives.

Recall how

$c' = 0$

$(x^n)' = nx^{n-1}$

$(c \cdot f(x))' = c \cdot f'(x)$

$(f(x) \pm g(x))' = f'(x) \pm g'(x)$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$(e^x)' = e^x$

in addition, we have the

product rule: $(f(x) \cdot g(x))' = f'(x)g(x) + f(x)g'(x)$

quotient rule: $\left(\dfrac{f(x)}{g(x)}\right)' = \dfrac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$

and the trignometric derivatives

$(\tan x)' = \sec^2 x$

$(\sec x)' = \sec x \tan x$

$(\csc)' = -\csc x \cot x$

$(\cot x)' = -\csc^2 x$

Example

1. Find the derivative

(a) $h(x) = 2\sqrt{x}\cos x$

$h'(x) = (2\sqrt{x}\cos x)'$ — we can factor out the constant 2

$= 2(\sqrt{x}\cos x)'$ — let $\sqrt{x} = f(x)$ and $\cos x = g(x)$ and use the product rule. $f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$ and $g'(x) = -\sin x$

PRODUCT RULE FORMULA

$h'(x) = f'(x)g(x) + f(x)g'(x)$

don't forget the 2! $h(x) = 2\left[\left(\frac{1}{2}x^{-\frac{1}{2}}\right)(\cos x) + (\sqrt{x})(-\sin x)\right]$

$$\boxed{h'(x) = 2\left[\frac{1}{2}x^{-\frac{1}{2}}(\cos x) + \sqrt{x}(-\sin x)\right]}$$