Derivatives

<u>Def.</u> (alternative) The <u>derivative</u> of f at the number a is

$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

*check other def by letting $x = a+h$ equivalence with

<u>Ex.</u> Find derivative using <u>only</u> the definition.

1) $f(x) = 3x+1$

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{3(x+h)+1-(3x+1)}{h} = \lim_{h\to 0} \frac{3x+3h-3x+1-1}{h} = \lim_{h\to 0} \frac{3h}{h} = 3$$

2) $f(x) = \sqrt{4+x}$

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\sqrt{4+x+h}-\sqrt{4+x}}{h} = \lim_{h\to 0} \frac{\sqrt{4+x+h}-\sqrt{4+x}}{h} \cdot \frac{\sqrt{4+x+h}+\sqrt{4+x}}{\sqrt{4+x+h}+\sqrt{4+x}}$$

$$= \lim_{h\to 0} \frac{4+x+h-(4+x)}{h\left(\sqrt{4+x+h}+\sqrt{4+x}\right)} = \lim_{h\to 0} \frac{h}{h\left(\sqrt{4+x+h}+\sqrt{4+x}\right)} = \frac{1}{\sqrt{4+x}+\sqrt{4+x}} = \frac{1}{2\sqrt{4+x}}$$

<u>Ex.</u> Suppose a billiard ball is dropped from height 100ft. Its height s at time t is given by $s = -16t^2 + 100$.

The ball hits the ground when $s = 0 = -16t^2 + 100$

$$16t^2 = 100$$
$$t^2 = \frac{100}{16}$$
$$t = \pm \frac{10}{4} = \pm\frac{5}{2} \to \frac{5}{2}$$ (since time should be positive)

<u>average velocity</u> of ball on time interval $[1,2]$ is $\frac{s(2)-s(1)}{2-1} = \frac{36-84}{2} = -48$ ft/sec

$[1,1.5]$ is $\frac{64-84}{0.5} = -40$ ft/sec

$[1,1.1]$ is $\frac{80.64-84}{0.1} = -33.6$ ft/sec

<u>instantaneous velocity</u> at $t=1$ is $\lim_{h\to 0} \frac{s(1+h)-s(1)}{h} = s'(1)$

$$s'(1) = \lim_{h\to 0} \frac{-16(1+h)^2+100-16(1)^2-100}{h} = \lim_{h\to 0} \frac{-16(1+2h+h^2)-16}{h} = \lim_{h\to 0} \frac{-32h-16h^2}{h} = \lim_{h\to 0} -32-16h$$

$$= -32$$