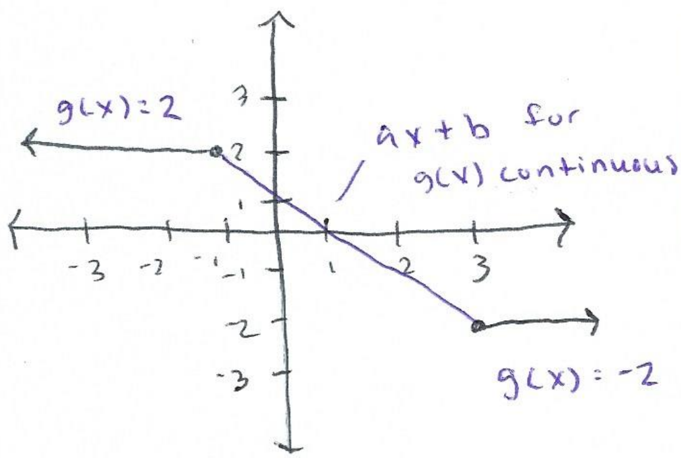
January 31, 2019

Examples

3. Find the constants a and b such that the function is continuous on the entire real number line.

$$g(x) = \begin{cases} 2, & x \le -1 \\ ax+b, & 1 < x < 3 \\ -2, & x \ge 3 \end{cases}$$

each part is continuous everywhere.



Remember,

g is continuous x = c if

1. g(c) is differentiable
2. $\lim_{x \to c} g(x)$ exists
3. $\lim_{x \to c} g(x) = g(c)$

we want g(x) to be continuous at the endpoints x = -1, x = 3

1. $g(-1) = 2$
2. $\lim_{x \to -1} g(-1) = 2$
3. $g(-1) = \lim_{x \to -1} g(-1) = 2$ ✓

using def 2.4.8 (pg 30)

$\lim_{x \to -1^+} g(x) = \lim_{x \to -1^+} (ax+b) = 2$

$2 = -a + b$ via direct sub. for x = -1.

1. $g(3) = -2$
2. $\lim_{x \to 3} g(3) = -2$ ✓
3. $g(3) = \lim_{x \to 3} g(3) = -2$

using def. 2.4.8

$\lim_{x \to 3^-} g(x) = \lim_{x \to 3^-} (ax+b) = -2$

$-2 = 3a + b$

We can solve the system of equations to find a, b.

$2 = -a + b \rightarrow b = 2 + a$

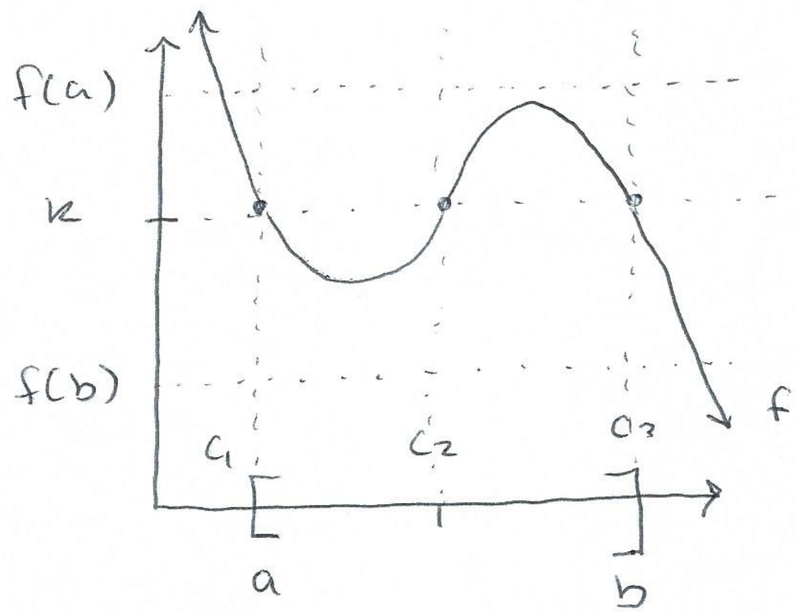
$-2 = 3a + b \rightarrow -2 = 3a + (2+a) \rightarrow -2 = 3a + 2 + a$

$-4 = 4a$. $\boxed{a = -1}$

so $b = 2 + (-1)$. $\boxed{b = 1}$

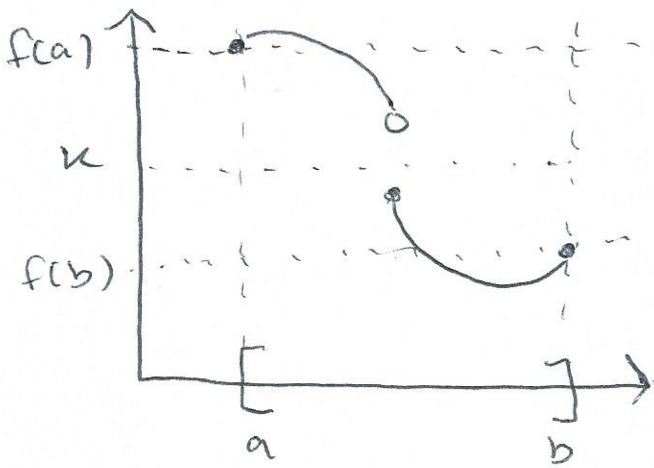
Theorem 2.4.15 Intermediate Value Theorem.

If f is continuous on the closed interval $[a, b]$, $f(a) \neq f(b)$, and k is any number between $f(a)$ and $f(b)$, then there is at least one number c in $[a, b]$ such that $f(c) = k$



f is shown to be continuous on the interval $[a, b]$ and $f(a) \neq f(b)$. Given k is a number between $f(a)$ and $f(b)$, then there is at least one number c such that $f(c) = k$. In this case, c_2 is between a and b, and $f(c_2) = k$.

If a function is not continuous, the Intermediate Value Theorem may not apply.



This graph shows jumps over the horizontal line $y = k$. So there is no value in $[a, b]$ to satisfy $f(c) = k$.

Example Use the Intermediate Value Theorem to show that the polynomial function $f(x) = x^3 + 2x - 1$ has zero in the closed interval $[0, 1]$. In other words, are there any x-values such that $f(x) = 0$?

Intermediate Value Theorem Requirements

- Is $f(x)$ continuous on $[0, 1]$? ✓
  Yes, because polynomial functions are continuous.
- Is it true that $f(0) \neq f(1)$? [The y-values of the end points are not the same)
  $f(0) = 0^3 + 2(0) - 1$ and $f(1) = 1^3 + 2(1) - 1$
  $f(0) = -1$ $\qquad$ $f(1) = 2$
  $-1 \neq 2$ and $f(0) \neq f(1)$. ✓
- Is $f(x) = 0$ $(y = 0)$ between $f(0)$ and $f(1)$?
  Yes, since $f(0) = -1$ and $f(1) = 2$, and 0 is between -1 and 2.

The Requirements for the Intermediate Value Theorem are satisfied, so by the Intermediate Value Theorem there is at least one c in $[0, 1]$ such that $f(c) = 0$.

2.5 Infinite Limits

Definition 2.5.1 A limit in which f(x) increases or decreases without bound as x approaches c is called an infinite limit.

Note 2.5.2 $\lim_{x \to c} f(x) = \infty$ does not mean that the limit exists. It really tells how the limit fails to exist by denoting the unbound behavior of f(x) as x approaches c.

Example 2.5.3 Determine the limit of the function $g(x) = \frac{1}{1-x}$ as x approaches 1 from the left and from the right.

Try to approach analytically with a table.

from left

x	0.9	0.99	0.999	0.9999	1
y	10	100	1000	10000	

$\lim_{x \to 1^-} \frac{1}{1-x} = \infty$ DNE

from right

x	1	1.0001	1.001	1.01	1.1
y		-10000	-1000	-100	-10

$\lim_{x \to 1^+} \frac{1}{1-x} = -\infty$ DNE

The graph shows