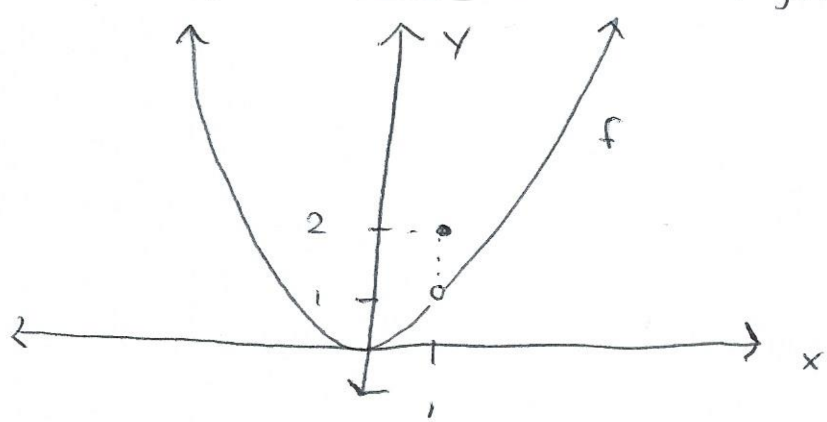
January 29, 2019

Check Blackboard for unit circle.

2.4 Continuity and one-sided limits

- A function f is continuous at c when the following are true:
  1. $f(c)$ is defined
  2. $\lim_{x \to c} f(x)$ exists
  3. $\lim_{x \to c} f(x) = f(c)$.

ex] Let $y = x^2$ such that it is graphed below.



If $c = 1$, what is true?

1. $f(c)$ is defined
   $f(1) = 2$. ✓

2. $\lim_{x \to c} f(x)$ exists
   $\lim_{x \to 1} x^2 = 1$. ✓

3. $\lim_{x \to c} f(x) = f(c)$.
   $\lim_{x \to 1} x^2 = 1$
   BUT
   $f(1) = 2$. X.

- A function is <u>continuous</u> on an <u>open interval</u> (a, b) when the function is continuous at each point in the interval.

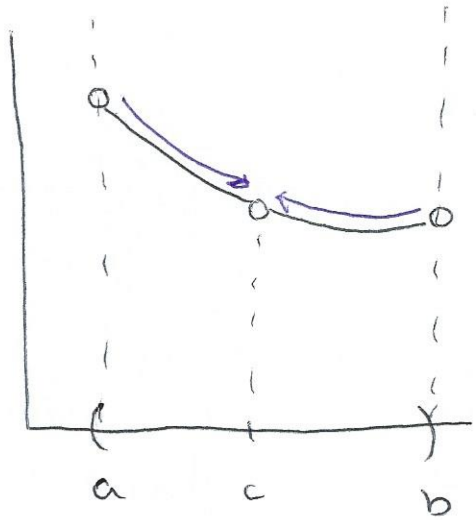
- A function that is continuous on the entire real number line $(-\infty, \infty)$ is everywhere continuous.

- If a function f is defined on I (except possibly c) and f is not continuous at c, then f is said to have a discontinuity at c.

- Discontinuities fall in two categories.

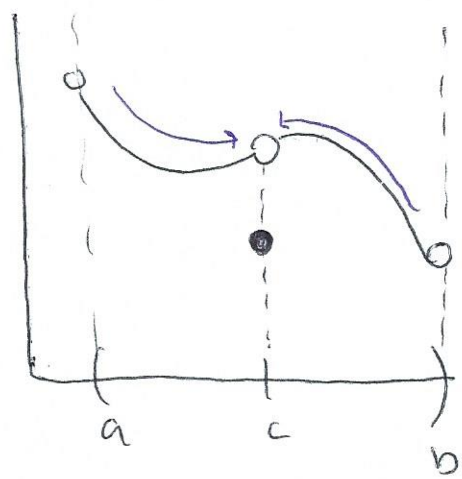
<u>(removable)</u> A discontinuity at c is called removable when f can be made continuous by approprvately defining (redefining $f(c)$). Otherwise, c is <u>(non-removable)</u>.

Examples



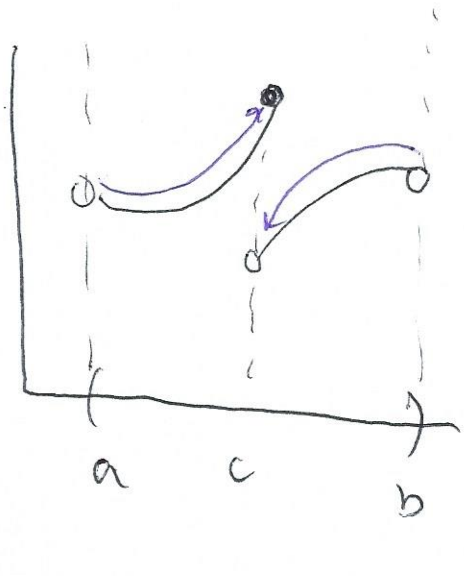
The left and right limits are the same value. They approach the discontinuity. If we put a point at $x = c$, the graph would be continuous.

This discontinuity at c, is removable.



Similar situation as above. The limit exists, and putting a point at $x = c$ would complete the graph as continuous.

At $x = c$, there is a removable discontinuity.



The left and right limits are different values. So the limit DNE. There is also a break (jump) between the two points on c.

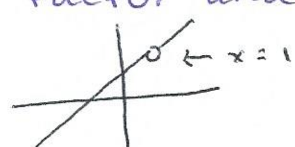
So at $x = c$, there is a non-removable discontinuity.

Another way to differentiate between removable and non-removable discontinuities...

1. If the function factors and the bottom term cancels, the discontinuity at the x-value for which the denominator was zero is removable, so the graph has a hole in it.

ex: $g(x) = \dfrac{x^2-1}{x-1}$ $x=1$ gives division by 0.

$g(x) = \dfrac{(x+1)\cancel{(x-1)}}{\cancel{x-1}}$ factor and cancel.



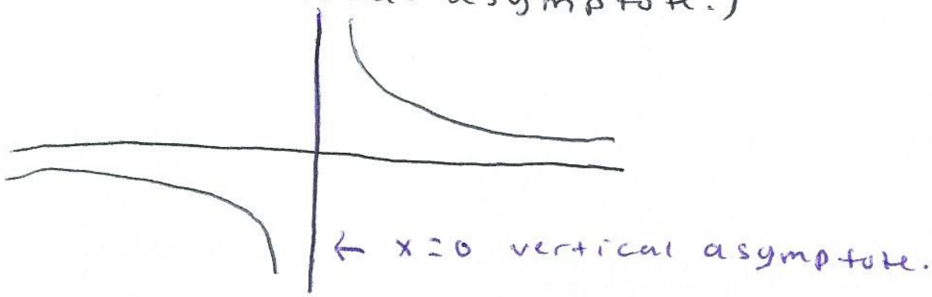
$g(x) = x+1.$

There is a removable discontinuity at $x=1$.

2. If a term does not cancel, the discontinuity at this x-value corresponding to this term for which the denominator is zero is nonremovable, and the graph has a vertical asymptote.

ex: $f(x) = \dfrac{1}{x}$ $x=0$ gives division by 0, but we cannot transform this to anything.

There is a non-removable discontinuity at $x=0$ (vertical asymptote.)



Examples Are the following continuous?

$$h(x) = \begin{cases} x+1 & x \le 0 \\ e^x & x > 0 \end{cases}$$

we use this formula if x is less than or equal to 0

we use this formula if x is greater than 0.

If $x \le 0$

$h(x) = x + 1$

we can add 1 to any x-value and still get a real number. So all values of x equal to and less than 0 are defined.

If $x > 0$

$h(x) = e^x$

e^x will be any real number if x is positive. So all values of x greater than 0 are defined.

Does the limit exist at $x = 0$?

Left of 0 is the negatives and we use $x+1$. We call this the left-hand limit.

$$\lim_{x \to 0^-} h(x) = \lim_{x \to 0} x + 1$$

$$= 0 + 1$$

$$\boxed{\lim_{x \to 0^-} h(x) = 1}$$

Right of 0 is the positives and we use e^x. This is the right-hand limit.

$$\lim_{x \to 0^+} h(x) = \lim_{x \to 0} e^x$$

$$= e^0 = 1$$

$$\boxed{\lim_{x \to 0^+} h(x) = 1}$$

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The left-hand and right-hand limits are both 1. Therefore the limit exists.

$$\boxed{\lim_{x \to 0} h(x) = 1}$$