Horizontal Asymptotes

Math 2413
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Suppose f is defined on (a,∞).

Def. $\lim_{x\to\infty} f(x) = L$ means that $f(x)$ can be made arbitrarily close to L by taking x sufficiently large.

Suppose f is defined on $(-\infty,a)$.

Def. $\lim_{x\to-\infty} f(x) = L$ means that $f(x)$ can be made arbitrarily close to L by taking x sufficiently large and negative.

*Note: Limit laws (except last three) still apply with a replaced by $\pm\infty$

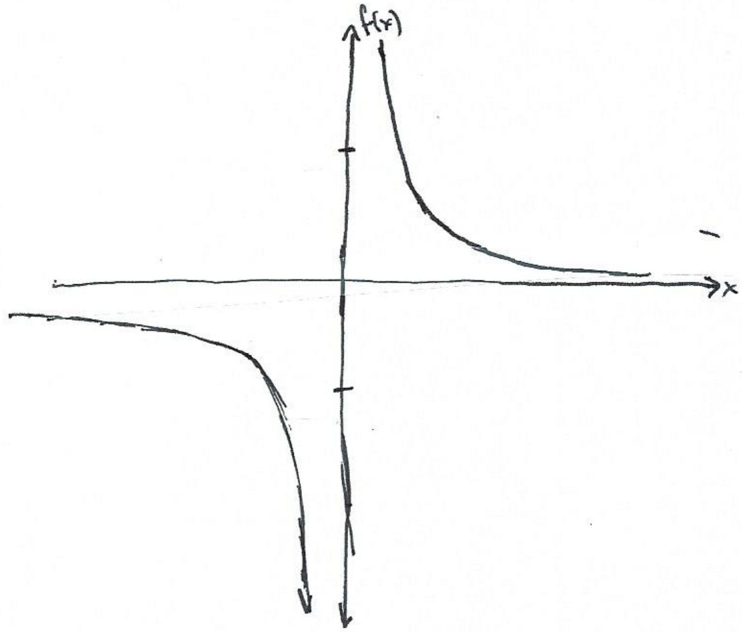
Ex. $f(x) = \frac{1}{x}$

x	$f(x)$
1	1
2	$1/2$
3	$1/3$
$\vdots$	$\vdots$
10,000	$\frac{1}{10{,}000} = 0.0001$
$\vdots$	$\vdots$

x	$f(x)$
1	1
$1/2$	2
$1/3$	3
$1/4$	4
$\vdots$	$\vdots$

x	$f(x)$
-2	$-\frac{1}{2}$
-4	$-\frac{1}{4}$
-8	$-\frac{1}{8}$
$\vdots$	$\vdots$

x	$f(x)$
-1	-1
$-1/3$	-3
$-1/6$	-6
$-1/9$	-9
$\vdots$	$\vdots$



$$\underbrace{\lim_{x\to\infty} f(x) = 0 \qquad \lim_{x\to-\infty} f(x) = 0}_{\text{Horizontal Asymptotes}}$$

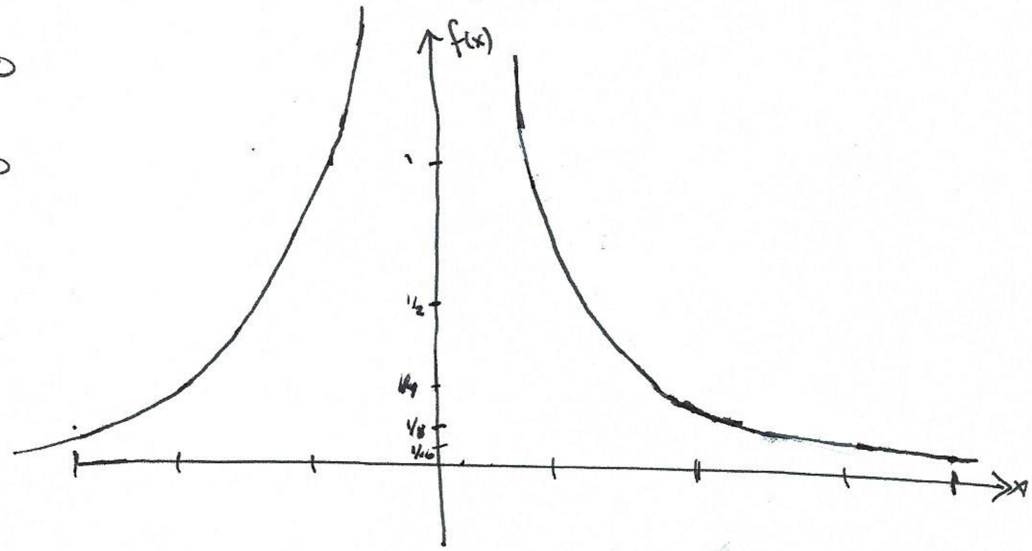
$$\underbrace{\lim_{x\to0^+} f(x) = \infty \qquad \lim_{x\to0^-} f(x) = -\infty}_{\text{Vertical Asymptotes}}$$

__Thm.__ If $r>0$ is rational, then $\lim\limits_{x\to\infty} \frac{1}{x^r} = 0$

If $r>0$ and x^r is defined for all x, then $\lim\limits_{x\to-\infty} \frac{1}{x^r} = 0$

__Ex.__ $r=2$: $\lim\limits_{x\to\infty} \frac{1}{x^2} = 0$

$\lim\limits_{x\to-\infty} \frac{1}{x^2} = 0$



__Ex__ $r=1/2$: $\lim\limits_{x\to\infty} \frac{1}{\sqrt{x}} = 0$

$\lim\limits_{x\to-\infty} \frac{1}{\sqrt{x}}$ DNE b/c $\sqrt{x}$ not real if $x<0$.

__Ex.__ Evaluate $\lim\limits_{x\to\infty} \frac{3x^2-x-2}{5x^2+4x+1}$ using limit laws

$$\left.\begin{aligned}\lim_{x\to\infty} 3x^2-x-2 &= \infty\\ \lim_{x\to\infty} 5x^2+4x+1 &= \infty\end{aligned}\right\} \to \text{"}\frac{\infty}{\infty}\text{" form}$$

$$\lim_{x\to\infty}\frac{3x^2-x-2}{5x^2+4x+1} = \lim_{x\to\infty}\frac{3x^2-x-2}{5x^2+4x+1}\cdot\underbrace{\frac{\frac{1}{x^2}}{\frac{1}{x^2}}}_{\text{b/c highest degree of top and bottom is 2}} = \lim_{x\to\infty}\frac{\frac{3x^2}{x^2}-\frac{x}{x^2}-\frac{2}{x^2}}{\frac{5x^2}{x^2}+\frac{4x}{x^2}+\frac{1}{x^2}} = \lim_{x\to\infty}\frac{3-\frac{1}{x}-\frac{2}{x^2}}{5+\frac{4}{x}+\frac{1}{x^2}} = \frac{\lim\limits_{x\to\infty}\left[3-\frac{1}{x}-\frac{2}{x^2}\right]}{\lim\limits_{x\to\infty}\left[5+\frac{4}{x}+\frac{1}{x^2}\right]}$$

$$= \frac{\lim\limits_{x\to\infty}(3) - \lim\limits_{x\to\infty}\left(\frac{1}{x}\right) - 2\lim\limits_{x\to\infty}\left(\frac{1}{x^2}\right)}{\lim\limits_{x\to\infty}(5) + 4\lim\limits_{x\to\infty}\left(\frac{1}{x}\right) + \lim\limits_{x\to\infty}\left(\frac{1}{x^2}\right)}$$

$$= \frac{3-0-2(0)}{5+4(0)+0} = \frac{3}{5}$$