Limit Laws

Math 2413
Dr. Kennedy
28 Jan 2019
pg8

Suppose that c is a constant and that $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ exist.

Then:

1) $\lim_{x\to a}\left[f(x) \pm g(x)\right] = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x)$

2) $\lim_{x\to a}\left[c\cdot f(x)\right] = c\cdot \lim_{x\to a} f(x)$

3) $\lim_{x\to a}\left[f(x)\cdot g(x)\right] = \left[\lim_{x\to a} f(x)\right]\cdot\left[\lim_{x\to a} g(x)\right]$

4) $\lim_{x\to a}\left[\dfrac{f(x)}{g(x)}\right] = \dfrac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$, $g(x) \neq 0$ in some interval around a (maybe not at a itself)

5) $\lim_{x\to a}\left[f(x)\right]^n = \left[\lim_{x\to a} f(x)\right]^n$, n a pos. int.

6) $\lim_{x\to a} c = c$

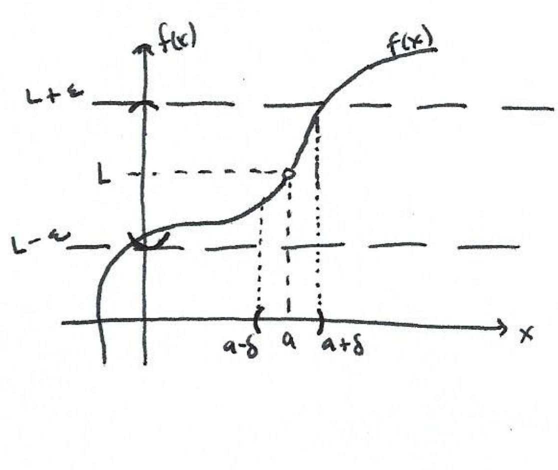
7) $\lim_{x\to a} x = a$

8) $\lim_{x\to a} x^n = a^n$, n a pos. int

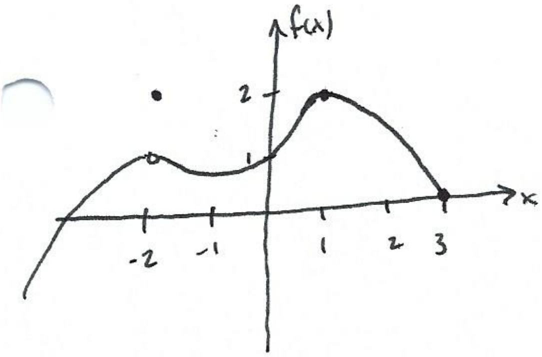
9) $\lim_{x\to a} \sqrt[n]{x} = \sqrt[n]{a}$, n a pos int
$a>0$ if n even

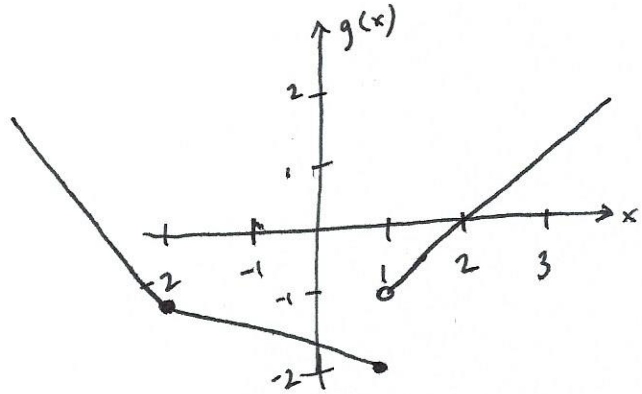
10) $\lim_{x\to a} \sqrt[n]{f(x)} = \sqrt[n]{f(a)}$

Def. $\lim_{x\to a} f(x) = L$ means that if $\varepsilon>0$, there is some $\delta>0$ such that if $0<|x-a|<\delta$, then $|f(x)-L|<\varepsilon$



Ex. Use limit laws to evaluate 1) $\lim_{x\to -2}\left[f(x)+5g(x)\right]$ if it exists





$\lim_{x\to -2} f(x) = 1$

$\lim_{x\to -2} g(x) = -1$

$$\lim_{x\to -2}\left[f(x)+5g(x)\right] = \lim_{x\to -2} f(x) + 5\lim_{x\to -2} g(x)$$

$$= 1 + 5(-1) = -4$$

2) $\lim_{x\to 1}\left[f(x)g(x)\right] = \left[\lim_{x\to 1} f(x)\right]\left[\lim_{x\to 1} g(x)\right]$ DNE b/c $\lim_{x\to 1^-} g(x) \neq \lim_{x\to 1^+} g(x)$

$\lim_{x\to -2}\left[f(x)g(x)\right] = \left[\lim_{x\to -2} f(x)\right]\left[\lim_{x\to -2} g(x)\right] = (+1)(-1) = -1$

3) $\lim_{x\to 2}\dfrac{f(x)}{g(x)}$ DNE b/c $\lim_{x\to 2} g(x) = 0$

$$\lim_{x\to -2}\frac{f(x)}{g(x)} = \frac{\lim_{x\to -2} f(x)}{\lim_{x\to -2} g(x)} = \frac{1}{-1} = -1$$

Ex. Evaluate the limits using limit laws

a) $\lim_{x\to 5} 2x^2-3x+4 = 2\cdot\lim_{x\to 5}(x^2) - 3\cdot\lim_{x\to 5}(x) + \lim_{x\to 5}(4) = 2\cdot\left(\lim_{x\to 5} x\right)^2 - 3(5) + 4$

$= 2(5)^2 - 15 + 4 = 50 - 11 = 39$

Thm functions continuous at every point in their domain include:

1) polynomial
2) rational
3) root
4) trig
5) inverse trig
6) exp
7) log