Math 2413
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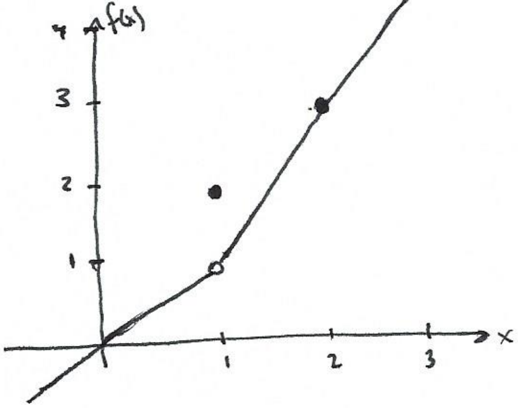
Ex. $f(x) = \begin{cases} x & \text{if } x<1 \\ 2 & \text{if } x=1 \\ 2x-1 & \text{if } x>1 \end{cases}$

a) graph f

b) Domain of f ?

c) Discuss the continuity of f



b) Domain of f is $\mathbb{R}$ = all real numbers

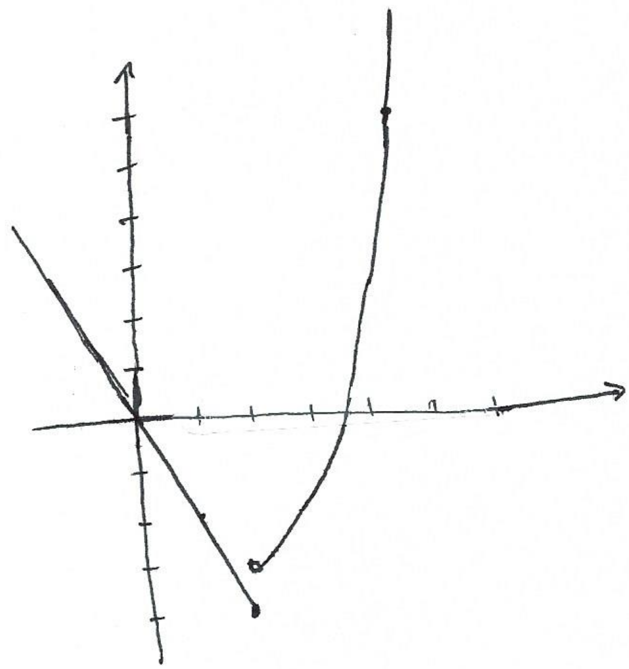
c) f is continuous at every number except 1

because at $a=1$:

$$\left.\begin{array}{l} \lim\limits_{x\to1^-} f(x) = \lim\limits_{x\to1^-} x = 1 \\ \lim\limits_{x\to1^+} f(x) = \lim\limits_{x\to1^+} 2x-1 = 1 \end{array}\right\} \longrightarrow \lim_{x\to1} f(x) = 1$$

But $\lim\limits_{x\to1} f(x) \neq f(1) = 2$

Ex. $f(x) = \begin{cases} -2x & \text{if } x\le 2 \\ x^2-4x+1 & \text{if } x>2 \end{cases}$



Domain $(f) = \mathbb{R}$

f is continuous everywhere except $x=2$

Ex. $f(x) = \dfrac{\sqrt{x}-3}{x-9}$, $x\neq 9$ $\mapsto$ $f(9)$ does not exist

$$\lim_{x\to9} \frac{\sqrt{x}-3}{x-9} = \lim_{x\to9} \frac{\sqrt{x}-3}{x-9}\cdot\frac{\sqrt{x}+3}{\sqrt{x}+3} = \lim_{x\to9} \frac{x-9}{(x-9)(\sqrt{x}+3)} = \lim_{x\to9} \frac{1}{\sqrt{x}+3} = \frac{1}{3+3} = \frac{1}{6}$$

$*\ (a+b)(a-b) = a^2-b^2$

Ex. $\lim_{h\to 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h\to 0} \frac{x^2 + h^2 + 2xh - x^2}{h} = \lim_{h\to 0} \frac{h^{\cancel{2}} + 2x\cancel{h}}{\cancel{h}} = \lim_{h\to 0} h + 2x = 2x$

Ex. $\lim_{x\to 2} \underbrace{\frac{\sqrt{x}+4}{x}}_{} = \frac{\sqrt{2}+4}{2}$

this function is continuous at $x=2$, so we substitute directly

Ex. $\lim_{x\to \pi/4} \frac{4x}{\tan x} = \frac{4(\frac{\pi}{4})}{1} = \pi$

Ex. $\lim_{x\to 1} \frac{\frac{1}{x} - 1}{x-1} = \lim_{x\to 1} \frac{\frac{1-x}{x}}{x-1} = \lim_{x\to 1} \frac{1}{x} \cdot \frac{1-x}{x-1} = \lim_{x\to 1} \frac{-1}{x} = -1$

$* \ \frac{1}{x} - 1 = \frac{1}{x} - 1 \cdot \frac{x}{x} = \frac{1}{x} - \frac{x}{x} = \frac{1-x}{x}$