<u>One-sided limits</u> — same as before, but we treat $x > a$ and $x < a$ separately

$$\lim_{x \to a^+} f(x) = L \qquad\qquad \lim_{x \to a^-} f(x) = L$$

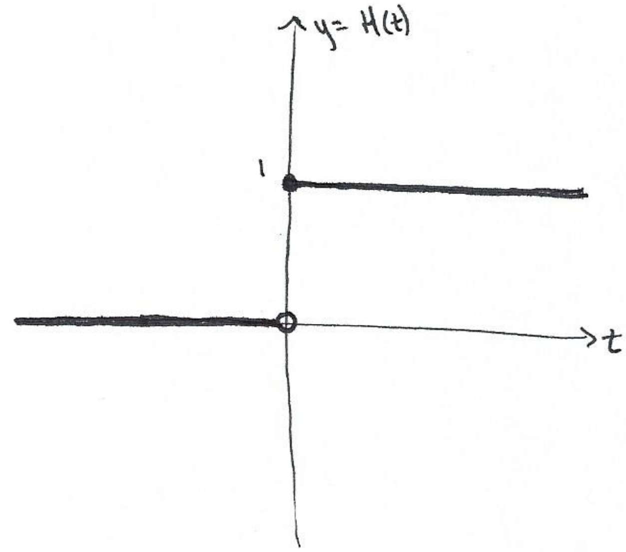
with $\underline{x > a}$, we can make $f(x)$ arbitrarily close to L by making x sufficiently close to a.

with $\underline{x < a}$, we can make $f(x)$ as close to L as we like by making x sufficiently close to a.

<u>Heaviside</u> function

$$H(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}$$

t	
-2	0
-1	0
0	1
1	1
2	1



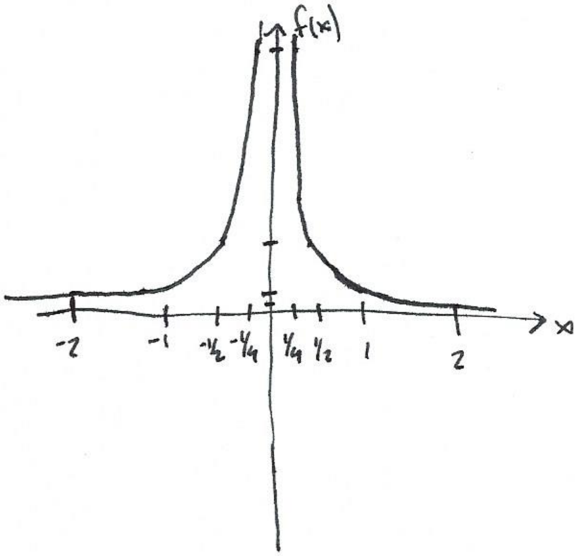
$$\lim_{t \to 0^-} H(t) = 0 \neq \lim_{t \to 0^+} H(t) = 1$$

<u>Prop:</u> $\lim_{x \to a} f(x) = L$ if and only if $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = L$.

<u>Ex</u> $f(x) = \frac{1}{x^2}$ Domain? – all real numbers except 0

Graph f.

x	-2	-1	$-1/2$	$-1/4$	$\frac{1}{4}$	$\frac{1}{2}$	1	2
$f(x)$	$\frac{1}{4}$	1	4	16	16	4	1	$\frac{1}{4}$



$\lim_{x \to 0} f(x) = \infty$, so the limit DNE

Continuity

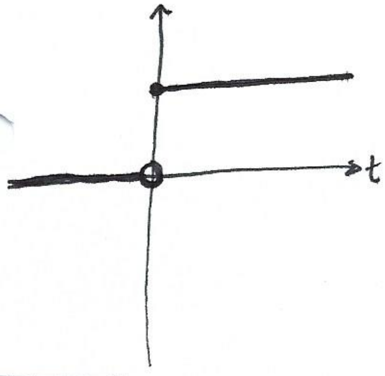
<u>Def.</u> The function f is <u>continuous</u> at the number a means:

1) $f(a)$ exists (a is in domain of f)

2) $\lim_{x \to a} f(x)$ exists

3) $\lim_{x \to a} f(x) = f(a)$

<u>Ex.</u> H(t)

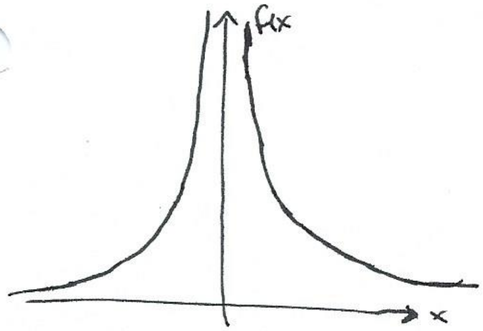


$H(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}$ is continuous at each $a \neq 0$.

H(t) is not continuous at $t = 0$ because

$\lim_{t \to 0^-} H(t) \neq \lim_{t \to 0^+} H(t)$, so $\lim_{t \to 0} H(t)$ DNE

Ex. $f(x) = \frac{1}{x^2}$



f is not continuous at $x = 0$ because $f(0)$ does not exist.

f is continuous at each $a \neq 0$, so f is continuous on its domain

Functions continuous on their domains:

- Polynomials : $a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0 = P(x)$
- Rational functions: $\frac{P(x)}{Q(x)}$
- power, root functions : x^n , $x^{1/n}$
- trig functions : $\sin(x)$, $\tan(x)$
- exponential, logarithmic functions : e^{-x} , $\log(x)$
- products, quotients, sums, differences, compositions of above

Ex. $\lim\limits_{x \to 1} \frac{x^2 - 1}{x - 1} = \lim\limits_{x \to 1} \frac{(x+1)\cancel{(x-1)}}{\cancel{(x-1)}} = \lim\limits_{x \to 1} x + 1 = 1 + 1 = 2$

*recall: when taking limits, x is not equal to a, just close to a.

Ex. Evaluate the limit

$$\lim_{x \to -4} \frac{\frac{1}{4} + \frac{1}{x}}{4 + x} = \lim_{x \to -4} \frac{\frac{x+4}{4x}}{4 + x} = \lim_{x \to -4} \frac{1}{4x} = \frac{-1}{16}$$