# January 22, 2019

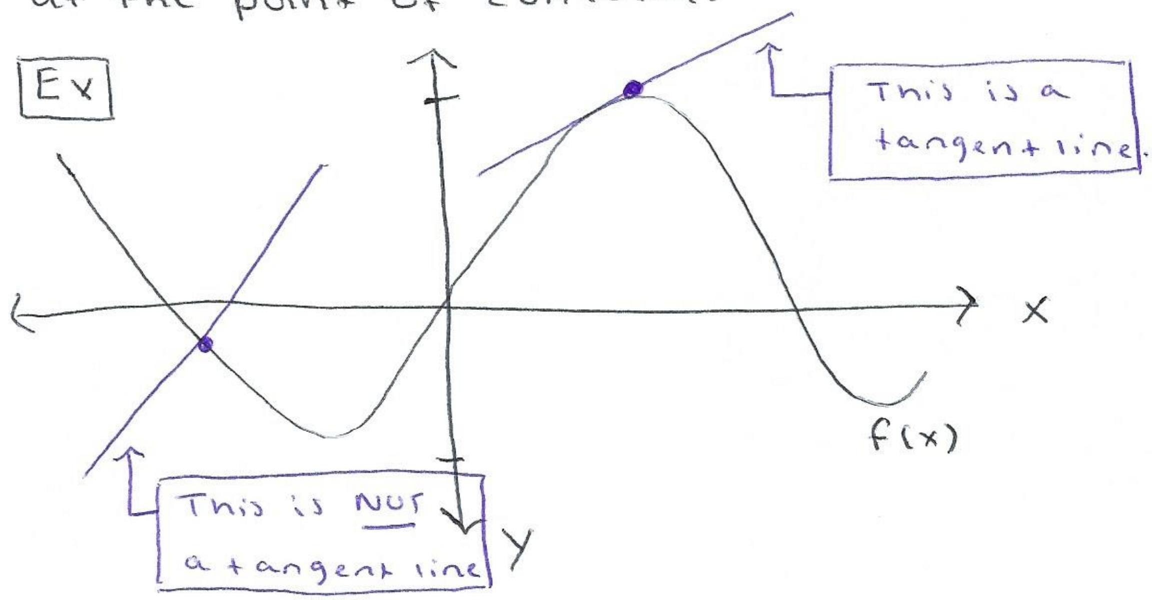


Chapter 2: Limits and Their Properties

- 2.1 : A Preview of Calculus

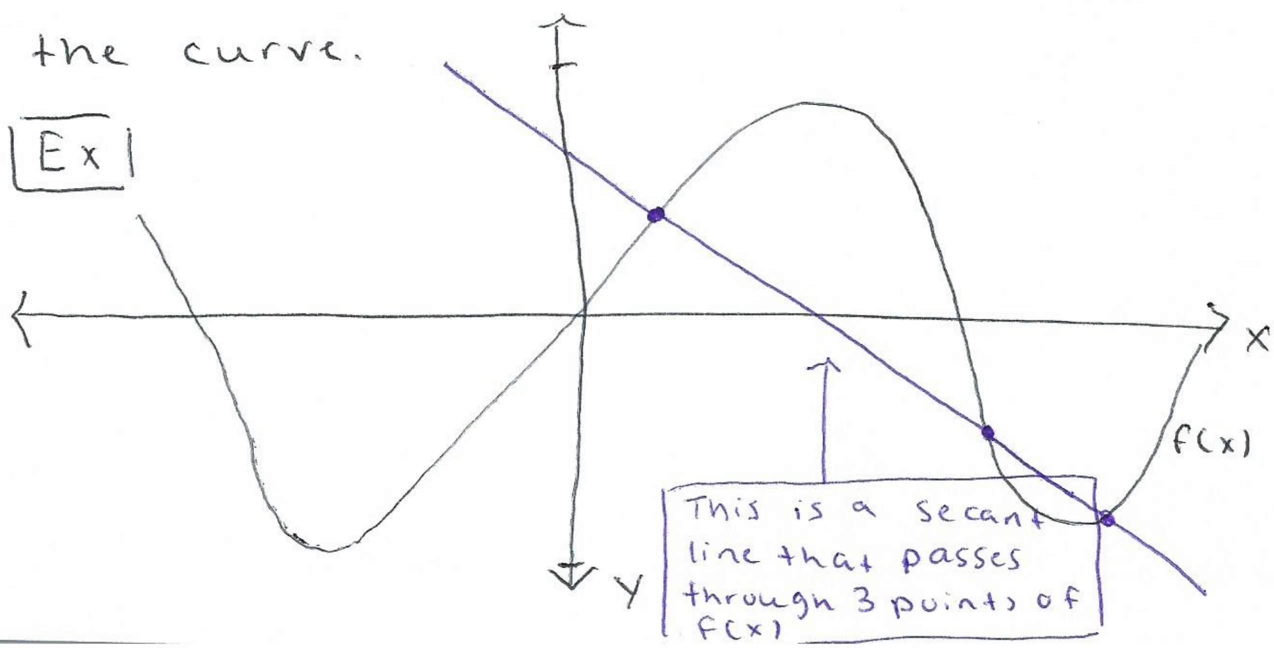
  - Def. 2.1.1
    A tangent line to a curve to a line that touches the curve. It should have the same direction of the curve at the point of contact.

Ex




A secant line passes through more than one points on the curve. It intersects the curve.

Ex



The slope of the tangent line is the limit of the slopes of the secant lines. The slope of the secant line through two points $P(a, f(a))$ and $Q(b, f(b))$ is

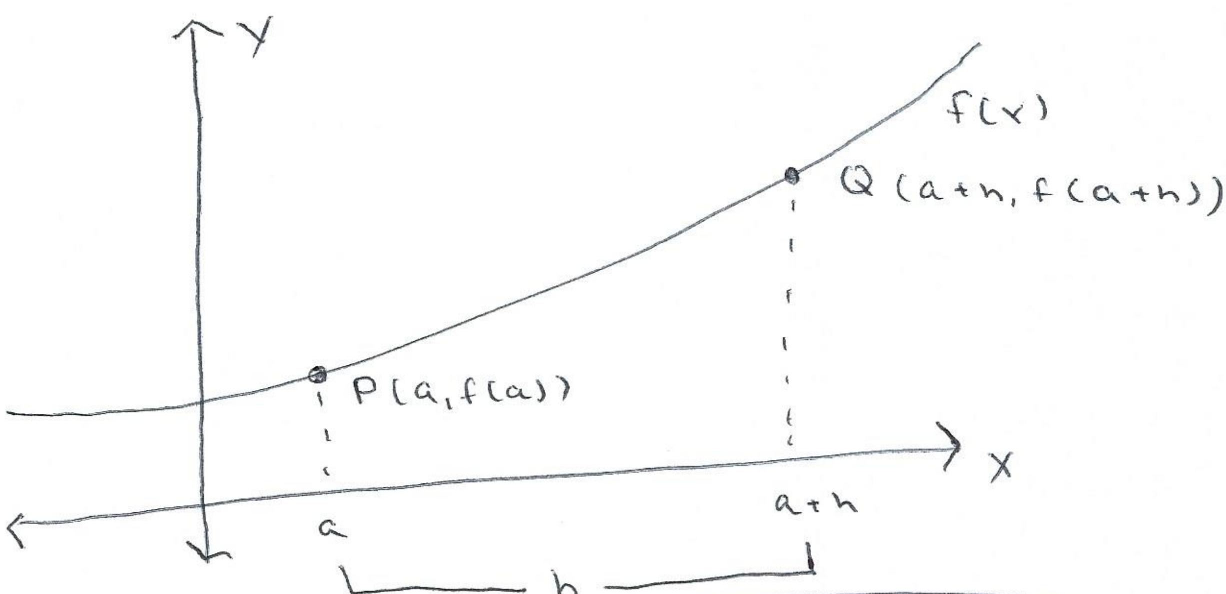
$$\boxed{m_{sec} = \frac{f(b) - f(a)}{b - a} \quad \text{or} \quad \frac{f(a) - f(b)}{a - b}}$$

In other words, the slope is the difference of the y-coordinates divided by the difference of the x-coordinates.

We may also use the formula

$$\boxed{m_{sec} = \frac{f(a+h) - f(a)}{h}}$$

between $P(a, f(a))$ and $Q(a+h, f(a+h))$. We call this the average rate of change between two points.



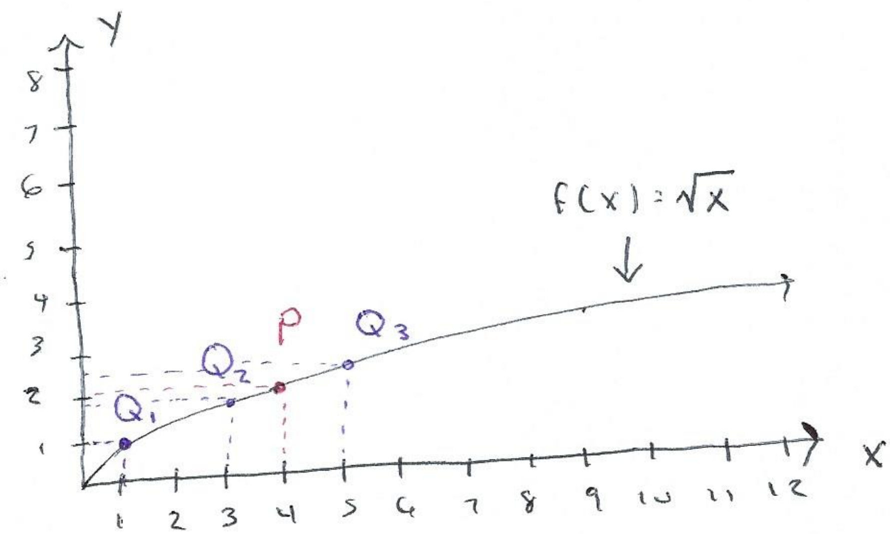

Since Q is a distance of h from a, we may represent it as a+h.

EX | 2.1.2 Calculate the function $f(x) = \sqrt{x}$ and the point $P(4, 2)$ on the graph of $f$.

① Graph $f = \sqrt{x}$ and the secant line through $P(4.2)$ and $Q(x, f(x))$ for x-values of 1, 3, and 5.

when

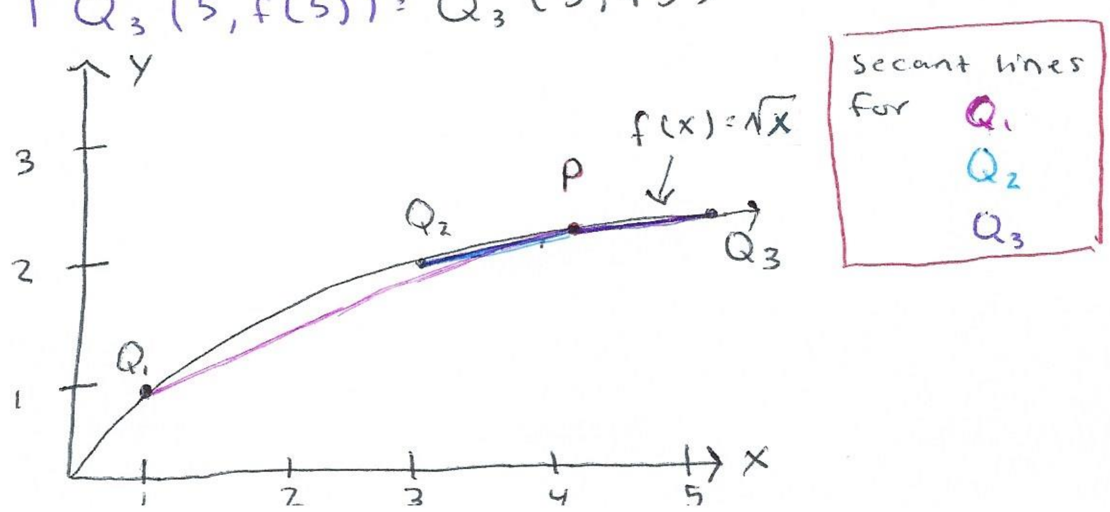
| $x =$ | 1 | 4 | 9 | 0 |
|---|---|---|---|---|
| $y =$ | 1 | 2 | 3 | 0 |




$Q_1(1, f(1)) = Q_1(1, \sqrt{1}) = Q_1(1, 1)$

$Q_2(3, f(3)) = Q_2(3, \sqrt{3})$

$Q_3(5, f(5)) = Q_3(5, \sqrt{5})$



(2) Find the slope of each secant line.

Formula : $m_{sec} = \dfrac{f(a) - f(b)}{a - b}$.

For $P(a, f(a))$ and $Q(b, f(b))$,

$m_{sec} = m_{PQ} = m_{QP}$

a) for $P(4,2)$ and $Q_1(1,1)$

$m_{PQ_1} = \dfrac{2-1}{4-1}$ $\qquad m_{Q_1P} = \dfrac{1-2}{1-4}$

$= \dfrac{1}{3}$ $\quad \underline{\underline{\text{ALSO}}} \quad = \dfrac{-1}{-3} = \dfrac{1}{3}$

$\boxed{\dfrac{1}{3} = m_{PQ_1} = m_{Q_1P}}$

b) for $P(4,2)$ and $Q_2(3, \sqrt{3})$

$m_{PQ_2} = \dfrac{2-\sqrt{3}}{4-3}$

$= \dfrac{2-\sqrt{3}}{1}$

$\boxed{m_{PQ_2} = 2-\sqrt{3}}$

c) for $P(4,2)$ and $Q_3(5, \sqrt{5})$

$m_{PQ_3} = \dfrac{2-\sqrt{5}}{4-5}$

$= \dfrac{2-\sqrt{5}}{-1} = -(2-\sqrt{5})$

$= -2+\sqrt{5}$

$\boxed{m_{PQ_3} = \sqrt{5} - 2}$

③ Use the results of part b) to estimate the slope of the tangent line to the graph of f at P(4,2). Describe how to improve the approximation of the slope.

Let's look at values approaching x=4 from the left and right sides.

| | | | | | |
|---|---|---|---|---|---|
| x | 3.99 | 3.999 | 4 | 4.001 | 4.01 |
| y | 2.32 | 2.12 | 2 | 2.19 | 2.21 |

as x gets nearer to 4, y approaches 2