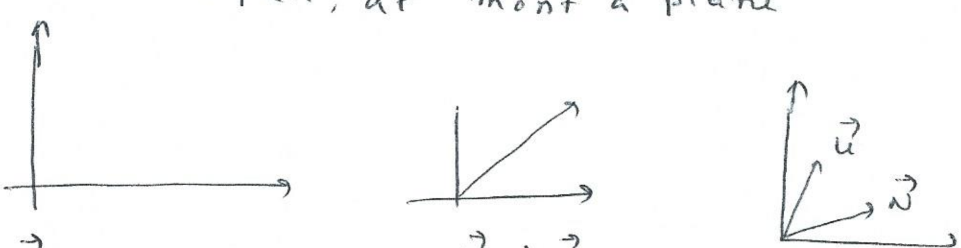
6th march, 2019

3.5 Linear independence

Def. A set of two or more vectors is linearly dependent if one of the vectors can be represented as a linear combination of others. A set containing only one vector is linearly dependent if that vector is zero vector.

A non empty set of vectors is linearly independent if it is not linearly dependent.

A set of vectors span, at most a plane



$\vec{u}, \vec{v} = 0$ $\qquad$ $\vec{u} = k\vec{v}$ for $k \in \mathbb{R}$ $\qquad$ $\vec{u} \neq k\vec{v}$ for all $k \in \mathbb{R}$

$\vec{u} = 0\vec{v}$ $\qquad$ $\vec{u}, \vec{v}$ are linearly independent

are linearly dependent

fact :

A set of vectors which contains the zero vector is linearly dependent.

The set $\{u, v\}$ is a linearly dependent set

consider, $\{\vec{0}, \vec{v}_1, \dots \vec{v}_n\}$

we can write

$\vec{0} = 0\vec{v}_1 + 0\vec{v}_2 + 0\vec{v}_3 \dots + 0\vec{v}_n$

Ex: The vectors $e_1 = i = \begin{bmatrix}1\\0\\0\end{bmatrix}$, $e_2 = j = \begin{bmatrix}0\\1\\0\end{bmatrix}$, $e_3 = \begin{bmatrix}0\\0\\1\end{bmatrix}$ are linearly independent.

suppose $\{\xi_1, \xi_2, \xi_3\}$ is linearly dependent.

then,

$$\xi_1 = a\xi_2 + b\xi_3$$

$$\hat{i} = a\hat{j} + b\hat{k}$$

$$\begin{bmatrix}1\\0\\0\end{bmatrix} = a\begin{bmatrix}0\\1\\0\end{bmatrix} + b\begin{bmatrix}0\\0\\1\end{bmatrix} = \begin{bmatrix}0\\a\\0\end{bmatrix} + \begin{bmatrix}0\\0\\b\end{bmatrix} = \begin{bmatrix}0\\a\\b\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}1\\0\\0\end{bmatrix} = \begin{bmatrix}0\\a\\b\end{bmatrix} \rightarrow \boxed{1 \neq 0}$$

Def

A set of vectors

$S = \{\vec{v}_1, \dots, \vec{v}_n\}$ is linearly independent if and only if

the only solution to $c_1\vec{v}_1 + c_2\vec{v}_2 + \dots + c_n\vec{v}_n = \vec{0}$ is the trivial

solution

$$c_1 = c_2 = \dots = c_n = 0$$

iff there are numbers $c_1 \dots c_n$ (not all zero) such that

$c_1\vec{v}_1 + c_2\vec{v}_2 \dots + c_n\vec{v}_n = 0 \rightarrow$ non trivial soln.

Ex:

Determine if the ~~set~~ vectors $v_1 = \begin{bmatrix}2\\4\\14\end{bmatrix}$ $V_2 = \begin{bmatrix}7\\-3\\15\end{bmatrix}$ $V_3 = \begin{bmatrix}-1\\4\\7\end{bmatrix}$

are linearly dependent or independent

Are there non-trivial soln to $c_1\vec{v}_1 + c_2\vec{v}_2 + c_3\vec{v}_3 = \vec{0}$

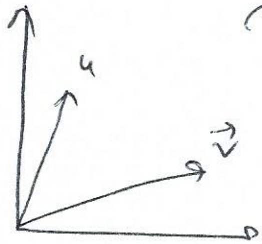
$$\Rightarrow \left[\begin{array}{ccc|c}2 & 7 & -1 & 0\\4 & -3 & 4 & 0\\14 & 15 & 7 & 0\end{array}\right] \rightarrow \begin{array}{ccc|c}2 & 7 & -1 & 0\\0 & -17 & 6 & 0\\0 & 0 & 2 & 0\end{array}$$

$\Rightarrow$ Triangular form of an homogeneous

system

$\Rightarrow$ only trivial soln

$\Rightarrow$ The vectors are LI.



$\rightsquigarrow$ vector space $V = \mathbb{R}^2$

Fact: A set of k ~~number of~~ vector in $\mathbb{R}^n$ is linearly dependent, if $k > n$.