$$\text{rank}(A) = \dim[RS(A)] = \dim[CS(A)] = 3$$

$$\dim[NS(A)] = 2$$

Linear algebra

27 th march

Example:

verify $rk(A) + \dim[NS(A)] = 5$

** find the bases for $RS(A)$, $CS(A)$, $NS(A)$

$$A = \begin{bmatrix} 1 & -2 & 3 & 0 & -1 \\ 2 & -4 & 7 & -3 & 3 \\ 3 & -6 & 8 & 3 & -8 \end{bmatrix} \xrightarrow[R_2: -2R_1+R_2]{R3=-3R_1+R_3} \begin{bmatrix} 1 & -2 & 3 & 0 & -1 \\ 0 & 0 & 1 & -3 & 5 \\ 0 & 0 & -1 & 3 & -5 \end{bmatrix}$$

$$\xrightarrow{R_3: -R_2+R_3} \begin{bmatrix} 1 & -2 & 3 & 0 & -1 \\ 0 & 0 & 1 & -3 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} = u$$

Basis $RS(A)$: $\left\{ \begin{bmatrix} 1 \\ -2 \\ 3 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ -3 \\ 5 \end{bmatrix} \right\}$

Basis for $CS(A)$ $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 7 \\ 8 \end{bmatrix} \right\}$

$\Rightarrow \text{rank}(A) = 2$

Basis for $NS(A)$

what do solutions to $u\bar{x} = \bar{0}$ look like?

$$u\bar{x} = \vec{0} \Rightarrow \left[\begin{array}{ccccc|c} 1 & -2 & 3 & 0 & -1 & 0 \\ 0 & 0 & 1 & -3 & 5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$x_1 - 2x_2 + 3x_3 + 0x_4 - x_5 = 0$$
$$x_3 - 3x_4 + 5x_5 = 0$$

$$\left.\begin{array}{l} x_1 + 3x_3 = 2x_2 + x_5 \\ x_3 = 3x_4 - 5x_5 \end{array}\right\} \quad \text{Let } x_2 = t_1,\ x_4 = t_2,\ x_5 = t_3$$

$$x_3 = 3t_2 - 5t_3$$

$$\Rightarrow x_1 + 3(3t_2 - 5t_3) = 2t_1 + t_3$$

$$x_1 = 2t_1 + t_3 - 9t_2 + 15t_3$$

$$x_1 = 2t_1 - 9t_2 + 16t_3$$

$$\vec{x}_1 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 2t_1 - 9t_2 + 16t_3 \\ t_1 + 0t_2 + 0t_3 \\ 0t_1 + 3t_2 - 5t_3 \\ 0t_1 + t_2 + 0t_3 \\ 0t_1 + 0t_2 + t_3 \end{bmatrix} = \begin{bmatrix} 2t_1 \\ t_1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -9t_2 \\ 0 \\ 3t_2 \\ t_2 \\ 0 \end{bmatrix} + \begin{bmatrix} 16t_3 \\ 0 \\ -5t_3 \\ 0 \\ t_3 \end{bmatrix}$$

$$\bar{x} = t_1\begin{bmatrix}2\\1\\0\\0\\0\end{bmatrix} + t_2\begin{bmatrix}-3\\0\\3\\1\\0\end{bmatrix} + t_3\begin{bmatrix}16\\0\\-5\\0\\1\end{bmatrix}$$

②

if $\vec{x} \in NS(A)$, then it is a linear combination of those 3 vectors,

Basis for NS(A)

$$\left\{ \begin{bmatrix}2\\1\\0\\0\\0\end{bmatrix}, \begin{bmatrix}-9\\0\\3\\1\\0\end{bmatrix}, \begin{bmatrix}16\\0\\-5\\0\\1\end{bmatrix} \right\}$$

square matrix

* Additions to theorem on pg 47 (page 209)

Let A be an mxn square matrix:

The following are equivalent

1) A is invertible

2) $A\bar{x} = \bar{b}$ has a unique solution for all b^n.

3) A is non singular: $A\bar{x} = \bar{0}$ has only the trivial soln.

4) $NS(A) = \{\vec{0}\}$

5) $\dim[NS(A)] = 0$

6) $\mathrm{rank}(A) = n$

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conditional statement

if A, then B

To prove; Assume A is true, and show that this means B must also be true.

if and only if

A if and only if B ⇒ if A, then B, and if B, then A

To prove:

① Assume A is true, then this implies B.

② " B is ", then " " A

*HW # 8, 9

8) Show that if $\{\bar{v}_1, \bar{v}_2, \bar{v}_3\}$ is linearly dependant, then $\{v_1, v_2, v_3, v_4\}$ is also.

Proof:

Suppose, $\{v_1, v_2, v_3\}$ is a linealy dependant set. So, we can write say $\bar{v}_3 = c_1 \bar{v}_1 + c_2 \bar{v}_2$ where $c_1, c_2 \in \mathbb{R}$.

Then, $\bar{v}_3 = c_1 \bar{v}_1 + c_2 \bar{v}_2 + 0 \vec{v}_4$

So, $\{\bar{v}_1, \bar{v}_2, \bar{v}_3, \bar{v}_4\}$ is also lin. dep.

Youtube: 3 blue 1 brown → Essence of linear algebra.

$A\vec{x} = \vec{b}$ = "Linear transformation" → $T(\vec{v})$
- special function