Section 3.4

Subspaces

Def: A subspace S of a vector space V is a subset of V which is itself a vector space.

Remember:

A vector space is a set whose elements are

1) closed under addition
2) closed under scalar multiplication
3) satisfy all 8 axioms on page 157

our vector space → $\mathbb{R}^n$

The set V is:

closed under addition

- if $\vec{u}, \vec{v} \in V$, then $\vec{u} + \vec{v} \in V$

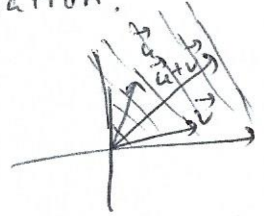
closed under scalar multiplication

- if $\vec{u} \in V$ and $r \in \mathbb{R}$, then $r\vec{u} \in V$

Ex.

Determine if

$S =$ 1st quadrant of $\mathbb{R}^2$ is closed under addition or scalar multiplication.



addition

Let $\vec{u}, \vec{v} \in S$

~~Sin~~

S is closed under

Scalar mult?

Let

$\vec{u} \in S$, Let, $s = -1$

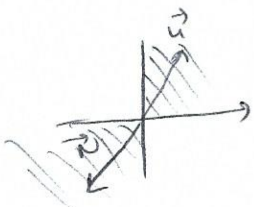
Then $-\vec{u} \notin S$

S is not closed under scalar multiplication

Determine if

$T =$ 1st and 3rd quadrant of $\mathbb{R}^2$

is closed under addition or scalar multiplication.



addition ?

Let $\vec{u}, \vec{v} \in S$. T is not closed under addition.

Scalar mult ?

T is closed under scalar multiplication

Ex: Let V be any vector space and let $S = \{\vec{0}\}$

S is closed under addition.

$\vec{0} + \vec{0} = \vec{0} \in S$

S is closed under scalar multiplication.

$\vec{0} \in S \quad r \in \mathbb{R}$

$r\vec{0} = \vec{0} \in S$

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you can check for yourself

all the axioms are satisfied

$S = \{\vec{0}\}$ is a subspace of any vector space.

$S =$ "zero vector space"

Theorem :

if a non-empty subset S of a vector space V is closed under addition and scalar multiplication, then it is a subspace of V.

let $V = M_{23}$ $\leadsto$ vector space of all 2x3 matrices.

let S = the set of all 2x3 matrices with second row = all zeros.

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is S a subspace of M_{23}?

closed under addition?

let $\vec{u}, \vec{v} \in S$, so, $u = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & 0 & 0 \end{bmatrix}$ $v = \begin{bmatrix} v_{11} & v_{12} & v_{13} \\ 0 & 0 & 0 \end{bmatrix}$

where $u_{11}, u_{12}, u_{13}, \dots v_{13} \in \mathbb{R}$

Then $u + v = \begin{bmatrix} u_{11}+v_{11} & u_{12}+v_{12} & u_{13}+v_{13} \\ 0 & 0 & 0 \end{bmatrix} \in S$

$\Rightarrow$ S is closed under addition

closed under scalar mult?

let $u \in S$ and $r \in \mathbb{R}$

$u = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & 0 & 0 \end{bmatrix}$ $r \in \mathbb{R}$

Then $ru = r[\quad] = \begin{bmatrix} ru_{11} & ru_{12} & ru_{13} \\ 0 & 0 & 0 \end{bmatrix}$

$\Rightarrow$ S is closed under scalar multiplication.

let $V = \mathbb{R}^2$

let S = the x-axis and the y axis

$S = \{(x, 0) \,||\, (0, y) \mid x, y \in \mathbb{R}\}$

closure under addition

let $u, v \in S$

let $u = (1,0)$, $v = (0,1)$, then

$u + v = \langle 1,1 \rangle \notin S$

S is not a subspace of $\mathbb{R}^2$

let $V = \mathbb{R}^2$

$S =$ x-y plane

$S = \{ (x,y,z) \in \mathbb{R}^3 \mid z = 0 \}$

<u>closure :</u>

let $u, v \in S$

$u = \langle u_{11}, u_{12}, 0 \rangle$, $v = \langle v_{11}, v_{12}, 0 \rangle$

$u + v = \langle u_{11} + v_{11}, u_{12} + v_{12}, 0 \rangle \in S$

1st thing to check for a subspace

Does the subset contain zero vector if not

it is not subspace.