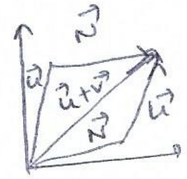
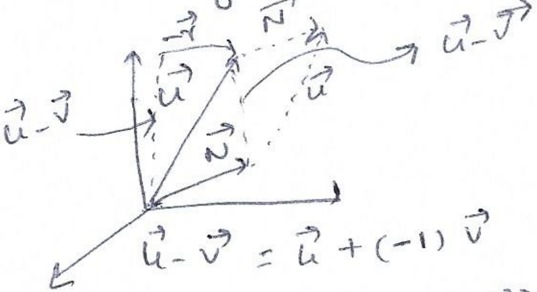
coordinates = components = entries of the vector representation when starting at the origin.





$\vec{u}+\vec{v}$

$$\vec{u}-\vec{v} = \vec{u}+(-1)\vec{v}$$
$$= \vec{u}+(-\vec{v})$$

Dot product and projection vector:

Dot product → scalar product

if $\vec{u} = (u_1, u_2, \ldots u_n)$ and $\vec{v} = (v_1, v_2, \ldots v_n)$

then, the dot product of $\vec{u}$ and $\vec{v}$ is:

$$\boxed{\vec{u}\cdot\vec{v} = u_1v_1 + u_2v_2 + \cdots + u_nv_n}$$

the magnitude (norm, length) of the vector

$\vec{u} = (u_1, u_2, \ldots u_n)$ in component form:

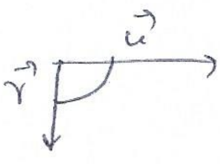
$$\|\vec{u}\| = \sqrt{u_1^2 + u_2^2 + \cdots + u_n^2}$$

alternate definition of the product

$$\vec{u}\cdot\vec{v} = \|\vec{u}\|\|\vec{v}\|\cos\theta$$

θ is the angle betn $\vec{u}$ and $\vec{v}$

The angle betn two vector $\vec{u}, \vec{v}$ is the angle θ determined by the ~~number~~ vector; $0 \le \theta \le \pi$



Ex. Let, $\vec{u} = (2, -1, 8, 7, 3)$ Find u, v and the angle between u and v

$\vec{v} = (-1, -8, -5, 1, 2)$

$$\vec{u} \cdot \vec{v} = 2(-1) + (-1)(-8) + 8(-5) + 7(1) + 3(2)$$

$$= \cancel{12} - 21$$

$$\vec{u} \cdot \vec{v} = \|\vec{u}\| \|\vec{v}\| \cos\theta$$

$$\cos\theta = \frac{\vec{u} \cdot \vec{v}}{u \, v}$$

$$\|u\| = \sqrt{4 + 1 + 64 + 49 + 3^2}$$

$$= \sqrt{127}$$

$$\|v\| = \sqrt{1 + 64 + 25 + 1 + 4}$$

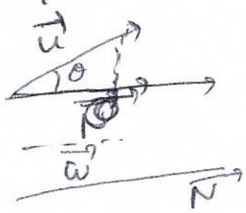
$$\|v\| = \sqrt{95}$$

$$\cos\theta = \frac{u \cdot v}{\|\vec{u}\| \|\vec{v}\|} = \frac{-21}{\sqrt{127}\sqrt{95}}$$

$$\theta = \arccos\left(\frac{-21}{\sqrt{127}\sqrt{95}}\right) \approx 101.02^\circ$$

<u>Projection</u>

The projection of the vector $\vec{u}$ onto $\vec{v}$



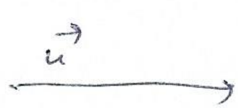
$\vec{w}$ = projection of $\vec{u}$ onto $\vec{v}$

A unit vector is a vector with magnitude = 1

A unit vector in the direction of a given vector, $\vec{v}$, is

$$\frac{\vec{v}}{\|\vec{v}\|}$$

u is a unit vector



$$\vec{w} = \|w\| \frac{v}{\|\vec{v}\|}$$

$$\cos\theta = \frac{\|w\|}{\|u\|}$$

$$\|w\| = \|u\| \cos\theta$$

$$\vec{w} = \|w\| \left(\frac{v}{\|v\|}\right)$$

$$= \|u\| \cos\theta \frac{v}{\|v\|}$$

$$\vec{u} \cdot \vec{v} = \|u\|\|v\| \cos\theta$$

$$\Rightarrow \|u\| \cos\theta = \frac{u \cdot v}{\|v\|}$$

$$= \left(\frac{u \cdot v}{\|v\|^2}\right) \vec{v}$$

$$\vec{w} = \frac{u \cdot v}{\|v\|^2} \vec{v} = \left(\frac{u + v}{\vec{v} \cdot \vec{v}}\right)$$

note

$$\vec{v} \cdot \vec{v} = \vec{v}\, v \cos 0$$

$$= v^2$$

Find the projection $\vec{w}$ of $\vec{u}$ onto $\vec{v}$:

$$\vec{u} = (2, 1, -3) \qquad \vec{v} = (-4, 1, -3)$$

$$\vec{w} = \left(\frac{u \cdot v}{\|v\|^2}\right) \vec{v}$$

$$u \cdot v = -8 + 1 + 9 = 2$$

$$v^2 = \sqrt{16 + 1 + 9} = (\sqrt{26})^2 = 26$$

$$\vec{w} = \frac{u \cdot v}{v^2} \vec{v} = \frac{2}{26} \vec{v} = \frac{1}{13} \vec{v}$$

$$= \left(-\frac{4}{13}, \frac{1}{13}, \frac{-3}{13}\right)$$

(4)

$$\vec{p} = \vec{u} - \vec{w} = (2, 1, -3) - \left(-\frac{4}{13}, \frac{1}{13}, \frac{-3}{13}\right) = \left(\frac{26+4}{13}, \frac{13-1}{13}, \frac{-39+3}{13}\right)$$

$$\vec{p} = \left(\frac{36}{12}, \frac{12}{13}, -\frac{36}{13}\right)$$

check: $p \perp \vec{v}$

$\vec{u} \cdot \vec{v} = 0 \Leftrightarrow \theta = 90°$

$$\vec{p} \cdot \vec{v} = \left(\frac{30}{13}\right)(-4) + \left(\frac{12}{13}\right)1 + \left(-\frac{36}{13}\right)(-3)$$

$$= \frac{-120 + 12 + 108}{13}$$

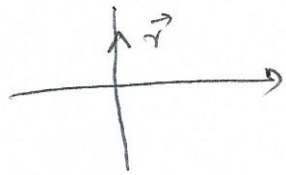
$$= \frac{0}{13}$$

$$= 0$$

what is the span of $\vec{v} = [0 \quad 2]$

$$\text{span}\{\vec{v}\} = y\text{-axis}$$

The span of a single vector is, at most a line.



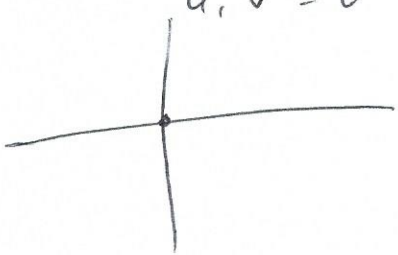
The span of two vectors, $\vec{u}$ and $\vec{v}$

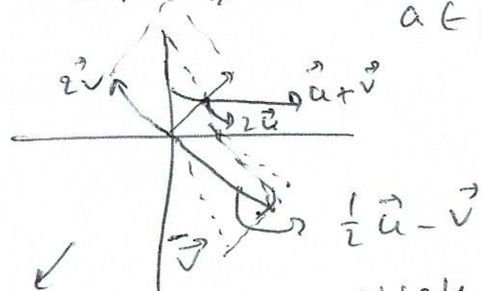
The span of $\vec{u}$ and $\vec{v}$ is the set of all "linear combination" of $\vec{u}$ and $\vec{v}$.

linear combination of $\vec{u}$ and $\vec{v}$ is:

$a\vec{u} + b\vec{v}$ where $a, b \in \mathbb{R}$

$\vec{u}, \vec{v} = \vec{0}$ $\qquad$ $\vec{u}, \vec{v} \neq \vec{0}$, but $\vec{u} = a\vec{v}$ $\quad a \in \mathbb{R}$





$\vec{u}, \vec{v} \neq \vec{0}$ and $\vec{u}$ is not scalar multiple of $\vec{v}$.

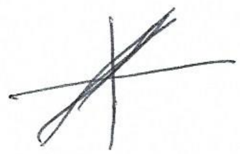
for 3 vector

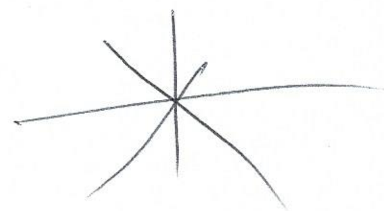
$a\vec{u} + b\vec{v} + c\vec{w} \qquad a, b, c \in \mathbb{R}$

$\bar{u}, \bar{v}, \bar{w} = 0$

$u, v, w \neq 0$ are scalar multiple of each other

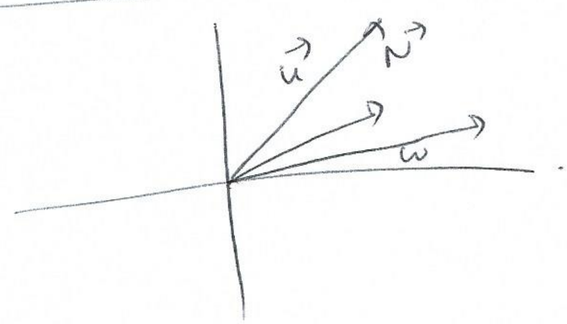
span$\{u, v, w\}$





2 of them are scalar multiple of each other

but the third one is not.



span$\{u, v, N\}$

a plane