3.1 : 3.2 vector spaces

A vector in euclidean n space is an ~~or~~ ordered n-tuple of real numbers $\vec{u} = (u_1, u_2, u_3 \cdots u_n)$ $\left\{ \underbrace{\overbrace{u_1, u_2 \cdots u_n \in \mathbb{R}}}_{\text{scalar}} \right\}$

n space is the collection of such possible n-tuples, denoted $\mathbb{R}^n$.

vector operations:

If $\vec{u}, \vec{v}$ are vectors in $\mathbb{R}^n$ and c is a scalar (real number), then

$$\vec{u} = (u_1 \cdots u_n)$$
$$\vec{v} = (v_1 \cdots v_n)$$

$$\vec{u} + \vec{v} = (u_1 + v_1, u_2 + v_2, \cdots u_n + v_n)$$

$$c\vec{u} = (cu_1, cu_2, \cdots, cu_n)$$

Examples:

Let $\left.\begin{array}{l}\vec{u} = (1, 2, 3, 4) \\ \vec{v} = (4, 3, 2, 1)\end{array}\right\} \rightarrow$ vectors in $\mathbb{R}^4$

$c = \frac{1}{2}$

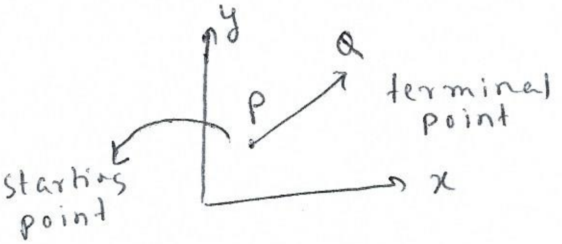
$$\vec{u} + \vec{v} = (1+4, 2+3, 3+2, 4+1)$$
$$= (5, 5, 5, 5)$$

$$c\vec{u} = \left(\frac{1}{2}, 1, \frac{1}{2}, 2\right)$$

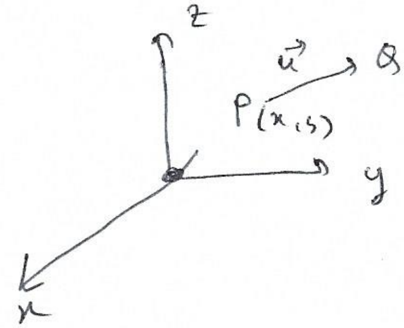
$\vec{u} - \vec{v} = \vec{u} + (-1)\vec{v} = (u_1 - v_1, \dots, u_n - v_n)$

2 Dimensions:

Represented as directed line segments from a starting point to a terminal point.



— Magnitude
— Direction



Magnitude of $\vec{u}$ is the distance betw start/terminal points

Notation. $\|\vec{u}\| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

~~$\|\vec{u}\| = \sqrt{\ }$~~

$\vec{u} = (u_1, u_2, \dots u_n)$

$\vec{u} = \vec{PQ}$, $P(p_1, \dots p_n)$, $Q(q_1, \dots q_n)$

$|\vec{u}| = \sqrt{(q_1 - p_1)^2 + (q_2 - p_2)^2 + \dots + (q_n - p_n)^2}$

properties of vectors:

Let, $\vec{u}, \vec{v}$ be vectors, r, s be scalars,

$\vec{u} + \vec{v} = \vec{v} + \vec{u}$

$(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$

$\vec{u} + \vec{0} = \vec{u}$ $\Rightarrow$ $\vec{u} + (-\vec{u}) = \vec{0}$

$\boxed{\vec{0} = (0, \dots, 0)}$
additive inverse of $\vec{u}$
$(-1)\vec{u} = -\vec{u}$

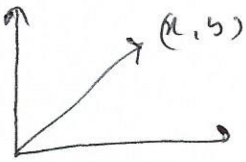
$\Rightarrow$ $(rs)\vec{u} = r(s\vec{u})$

Properties ~~of vectors~~:

~~Let, $\vec{u}, \vec{v}, \vec{w}$ be vector, r, s be scalars~~:

~~$\vec{u} + \vec{v} = \vec{v} + \vec{u}$~~

2 Dimensions

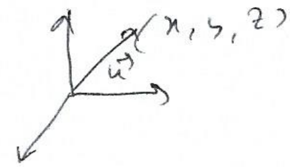


usually, we let the starting point be the origin: $(0, 0, \dots 0)$

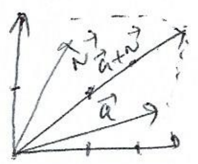
$\vec{u} = (x, y)$

$\uparrow$

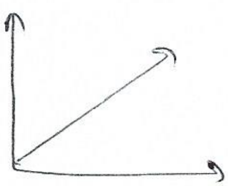
components



Let, $\vec{u} = (2, 1)$, $\vec{v} = (1, 3)$ $\vec{u} + \vec{v} = (3, 4)$



Scalar multiplication:



$\vec{u}\,(2, 1)$

$2\vec{u} = (4, 2)$ → same direction as $\vec{u}$, twice as long

$\frac{1}{2}\vec{u} = (1, \frac{1}{2})$ → same direction as $\vec{u}$, but half as long.

$-\vec{u} = (-2, -1)$ → same size as $\vec{u}$, in opposite direction.