Def: A matrix in in echelon form if:

a) all rows containing only zeros are grouped at the bottom of the matrix.

b) for each row that does not contain only zeros, the pivot appears strictly to the right of the pivot of each row that lies above it.

Def: A pivot in the first non-zero entry in any row of a matrix.

Gaussian elimination:

$$2x_1 - x_2 - x_3 = 3$$
$$-6x_1 + 6x_2 + 5x_3 = -3$$
$$4x_1 + 4x_2 + 7x_3 = 3$$

Coefficient matrix | matrix of constants

$$\left[\begin{array}{ccc} 2 & -1 & -1 \\ -6 & 6 & 5 \\ 4 & 4 & 7 \end{array}\right] \quad \left[\begin{array}{c} 3 \\ -3 \\ 3 \end{array}\right]$$

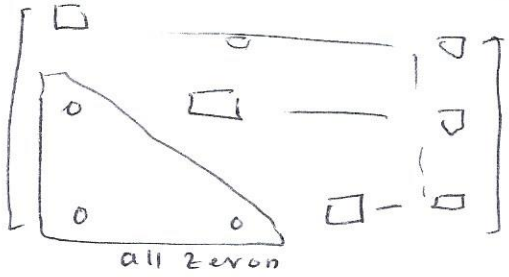
augmented matrix

$$\left[\begin{array}{ccc|c} 2 & -1 & -1 & 3 \\ -6 & 6 & 5 & -3 \\ 4 & 4 & 7 & 3 \end{array}\right]$$

Perform finite sequence of row operations,

① mult a row by non zero const
② interchange any two row ③ add a non-zero mult. of one row to another.

→ try to trasform into triangular form of eehelon form



eehelon form Ex.

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

pivots

$$\begin{bmatrix} \boxed{2} & 1 & 0 & 0 \\ 0 & \boxed{1} & 2 & 3 \\ 0 & 0 & 0 & \boxed{1} \end{bmatrix}$$

$$\begin{bmatrix} 0 & 2 & 1 & 5 & 5 \\ 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Not eehelon

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 2 & 3 & 5 \\ 0 & 1 & 3 & 0 \end{bmatrix} \longrightarrow \begin{bmatrix} 0 & 1 & 3 & 0 \\ 1 & 2 & 3 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 3 & 5 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Echelon

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 0 & \boxed{3} & 2 & 1 \\ 0 & 0 & \boxed{1} & 2 & 3 \end{bmatrix}$$

→ Gaussian elimination

$$\begin{cases} x_1 - 2x_2 - 3x_3 = -1 \\ x_1 - x_2 - 2x_2 = 1 \\ -x_1 + 3x_2 + 5x_3 = 2 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -3 & -1 \\ 1 & -1 & -2 & 1 \\ -1 & 3 & 5 & 2 \end{array}\right] \xrightarrow{R_3 : R_1 + R_3} \left[\begin{array}{ccc|c} 1 & -2 & -3 & -1 \\ 1 & -1 & -2 & 1 \\ 0 & 1 & 2 & 1 \end{array}\right] \xrightarrow{R_2 : -R_1 + R_2}$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -3 & -1 \\ 0 & 1 & 1 & 2 \\ 0 & 1 & 2 & 1 \end{array}\right] \xrightarrow{R_3 : -1R_2 + R_3} \left[\begin{array}{ccc|c} 1 & -2 & -3 & -1 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

↳ triangular form

*** Do back substitution

$$x_1 - 2x_2 - 3x_3 = -1$$

$$x_2 + x_3 = 2$$

$$\boxed{x_3 = -1}$$

$$x_2 - 1 = 2$$

$$\Rightarrow \boxed{x_2 = 3}$$

$$x_1 - 2(3) - 3(-1) = -1$$

$$\Rightarrow x_1 = 2$$

unique solution

$$\boxed{(x_1, x_2, x_3) = (2, 3, -1)}$$

$$x_1 + 4x_2 - 3x_3 = 2$$

$$3x_1 - 2x_2 - x_3 = -1$$

$$-x + 10x_2 - 5x_3 = 3$$

$$\left[\begin{array}{ccc|c} 1 & 4 & -3 & 2 \\ 3 & -2 & -1 & -1 \\ -1 & 10 & -5 & 3 \end{array}\right] \xrightarrow[R_2 = -3R_1 + R_2]{R_3 \to 1R_1 + R_3} \left[\begin{array}{ccc|c} 1 & 4 & -3 & 2 \\ 0 & -14 & 8 & -7 \\ 0 & 14 & -8 & 5 \end{array}\right] \xrightarrow{R_3 : 1R_2 + R_3}$$

$$\left[\begin{array}{ccc|c} 1 & 4 & -3 & 2 \\ 0 & -14 & 8 & -7 \\ 0 & 0 & 0 & -2 \end{array}\right]$$

Echelon form

$$x_1 + 4x_2 - 3x_3 = 2$$

$$-14x_2 + 8x_3 = -7$$

$$\boxed{0 = -2}$$

NO solution