A system of linear equations (Linear system) is a finite collection of linear equation:

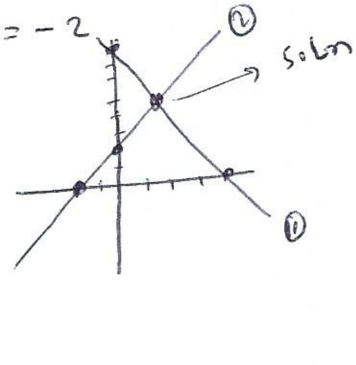
$$a_{11}x_1 + a_{12}x_2 + \cdots a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots a_{2n}x_n = b_2$$
$$\vdots$$
$$a_{m1}x_1 + a_{m2}x_2 + \cdots a_{mn} = b_n$$

A solution is a linear system is a sequence of numbers $t_1, t_2 \cdots t_n$ which solves each equation simultaneously.

A linear system with at least one solution is called consistent. Otherwise, it is called inconsistant.

Simple example:

$$\begin{array}{ll} ① & 2x + y = 8 \\ ② & x - y = -2 \end{array} \quad \text{or} \quad \begin{array}{l} 2x_1 + x_2 = 8 \\ x_1 - x_2 = -2 \end{array}$$



Substitution

$$\begin{array}{l} 2x + y = 8 \\ x - y = -2 \end{array} \Big| \Rightarrow x = -2 + y$$

$\Rightarrow 2(-2+y) + y = 8$

$\Rightarrow -4 + 3y = 8$

$\Rightarrow y = 4$

$x - 4 = -2$

$\Rightarrow x = 2$

Elimination

$$2x + y = 8$$
$$x - y = -2$$

$$3x = 6$$
$$\Rightarrow x = 2$$
$$2 - y = -2$$
$$\Rightarrow y = 4$$

** $\boxed{\begin{aligned} 6x_1 - 10x_2 &= 0 \\ -3x_1 + 5x_2 &= 8 \end{aligned}}$ → parallel lines

Elimination :

$$6x_1 - 10x_2 = 0$$
$$-6x_1 + 10x_2 = 16$$

$$0 + 0 = 16$$
$$0 = 16$$

No solution

$\boxed{\begin{aligned} 4x_1 + 10x_2 &= 14 \\ -6x_1 - 15x_2 &= -21 \end{aligned}}$ → multiply by $\frac{3}{2}$

→ same line

Elimination

$$6x_1 + 15x_2 = 21$$
$$-6x_1 - 15x_2 = -21$$

$$0 + 0 = 0$$

infinitly many solution

Soln set

$$4x_1 + 10x_2 = 14$$
$$-6x_1 - 15x_2 = -21$$

$$4x_1 + 10x_2 = 14$$
$$x_1 = \frac{14 - 10x_2}{4} = \boxed{\frac{7 - 5x_2}{2} = x_1}$$

Let, $x_2 = t$, then

$$x_1 = \frac{7 - 5t}{2}$$

$$\left\{ \left(\frac{7-5t}{2}, t \right) \mid t \in \mathbb{R} \right\}$$

A linear system has either:

1. No solution
2. A unique solution
3. infinitly many solution

$2x + 3y - z = 2$

$6x + 2y + z = 1$

$x - y + 2z = 5$

$x - 2x_2 - 5x_3 + 3x_4 = 2$

$x_2 + 3x_3 - 4x_4 = 7$ → Trianguar form

⇓

$x_3 + 2x_4 = -4$ has a unique soln.

$\boxed{x_4 = 5}$

$\Rightarrow x_3 + 2 \cdot 5 = -4$

$\Rightarrow \boxed{x_3 = -14}$

$\Rightarrow x_2 + 3(-14) - 4 \cdot 5 = 7$

$\Rightarrow \boxed{x_2 = \cancel{55}\ 69}$

$\Rightarrow x_1 - 2 \cdot 69 - 5 \cdot (-14) + 3 \cdot 5 = 2$

$\Rightarrow \boxed{x_1 = 55}$

Solution $(55, 69, -14, 5)$

$3y + 2z = 7$

$x + 4y - 4z = 3$

$3x + 3y + 8z = 1$

Elementary operation

① . add a multiple of one equation to other.

② interchange two equations.

③ multiply an eqn. by a nonzero constant.

Matrix of coefficients

$$\begin{bmatrix} 0 & 3 & 2 \\ 1 & 4 & -4 \\ 3 & 3 & 8 \end{bmatrix}$$

matrix of constants

$$\begin{bmatrix} 7 \\ 3 \\ 1 \end{bmatrix}$$

Augmented matrix

$$\left[\begin{array}{ccc|c} 0 & 3 & 2 & 7 \\ 1 & 4 & -4 & 3 \\ 3 & 3 & 8 & 1 \end{array}\right]$$